Problems and Results Motivated by Efficient Computation of the Independence Number

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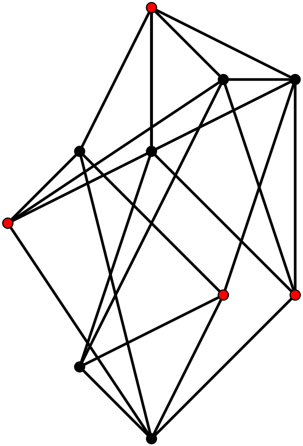
I. Introduction, Applications, Complexity

The Independence Number of a Graph



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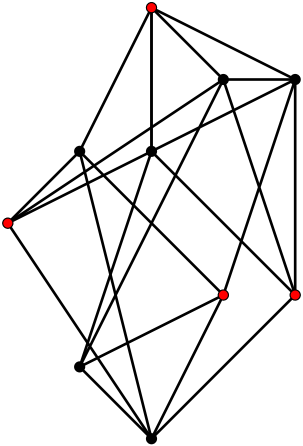

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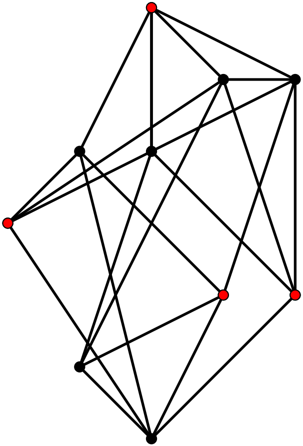
The Independence Number of a Graph



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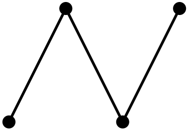
$$\alpha = 4.$$

Independent Sets and Chemical Properties



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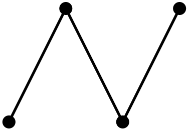
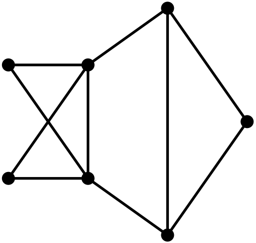


- The number of independent sets σ correlates both with alkane heats of formation and boiling points.

• R. Merrifield and H. Simmons, The Structure of Molecular Topological Spaces, *Theoretica Chimica Acta*, 1980.

The matching number of a graph



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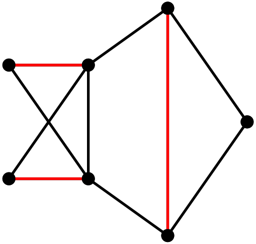
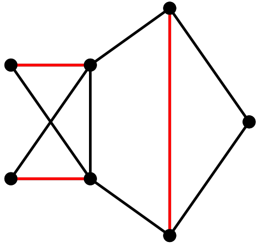


Let M=red.

The matching number of a graph



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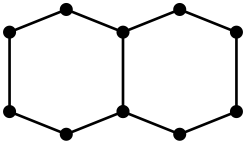
M is a maximum matching,

and $\mu = 3$.

Molecular Stability



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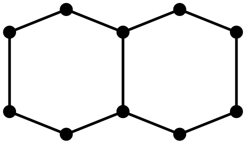

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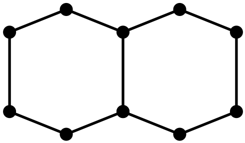


- Stable benzenoids maximize their matching number
- and minimize their independence number.

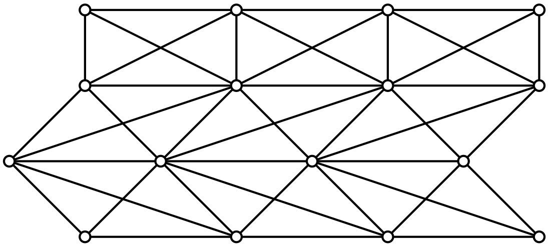
• R. Pepper, An upper bound on the independence number of benzenoid systems, *Discrete Applied Mathematics*, 2008.

Which Fullerene Isomers are Stable?

Atoms	Isomer	# of Isomers	α	Rank	Max	Min
60	C_{60}:1 (I_h)	1812	24	1	28	24
70	C_{70}:1 (D_{5h})	8149	29	1	33	29
76	C_{76}:1 (D_2)	19151	32	1	36	32
78	C_{78}:1 (D_3)	24109	33	1 (3)	37	33
	C_{78}:3 (C_{2v})		34	2		
	C_{78}:2 (C_{2v})		33	1 (3)		
84	C_{84}:22 (D_2)	51592	36	1 (17)	40	36
	C_{84}:23 (D_{2d})		36	1 (17)		

- S. Fajtlowicz, and C. E. Larson, Graph-theoretic Independence as a Predictor of Fullerene Stability, *Chemical Physics Letters*, 2003.

Shannon Capacity



- The zero-error capacity of a alphabet is lim $\sqrt[n]{\alpha(G^n)}$.

• C. Shannon, The zero error capacity of a noisy channel, *IRE Transactions on Information Theory*, 1956.

Optimal Communication Networks

• G. Brinkmann, S. Crevals, J. Frye, An independent set approach for the communication network of the GPS III system, *Discrete Applied Mathematics*, 2013.

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- Matching number: $n - 2\mu \leq \alpha \leq n - \mu$.
- Clique Number: $\alpha(G) = \omega(\bar{G})$.
- Covering Number: $\alpha = n - \tau$.

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- Östergård's Cliquer algorithm is a simple, fast and popular practical general algorithm.
- New general algorithms may be faster - San Segundo's BBMC.

• J. M. Robson, Algorithms for Maximum Independent Sets, Journal of Algorithms 7 (1986) 425–440.
• P. Östergård, A fast algorithm for the maximum clique problem, Discrete Applied Mathematics 120 (2002) 197–207.
• P. San Segundo, An improved bit parallel exact maximum clique algorithm, *Optimization Letters*, 2011.

Independence number is NP-hard

The Independent Set Decision Problem:

Given a graph G and an integer k, does G have an independent set of size at least k?

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• R. M. Karp, Reducibility Among Combinatorial Problems, Complexity of Computer Computations, 1972, 85–103.
• M. Garey and D. Johnson, Computers and Intractability, W. H. Freeman and Company, New York, 1979.

Does P=NP?

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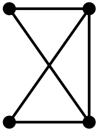
"My hunch is that P=NP, contrary to general belief."

• B. Bollobás, The Future of Graph Theory, Quo Vadis, Graph Theory?, 1993, 5–11 .

II. A Structural Result

A König-Egervary graph (or KE graph) is a graph where $\alpha + \mu = n$.



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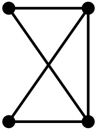

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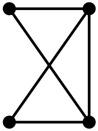

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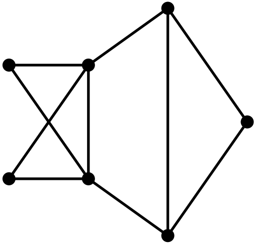
The critical difference d is the maximum value of $|I| - |N(I)|$, for all independent sets I. An independent set I_c which realizes d is a critical independent set.



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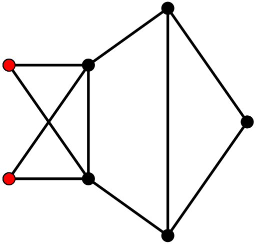

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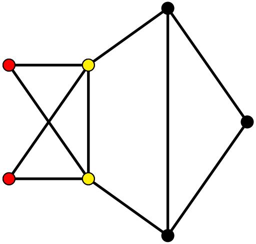

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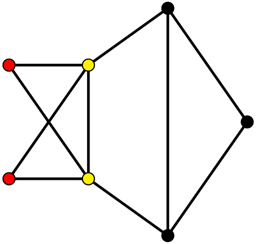

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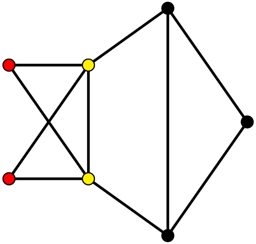

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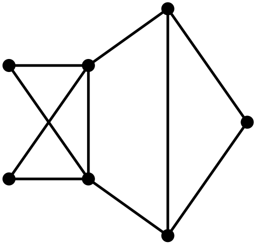
$d = 0$ and I_c is a critical independent set.

A maximum critical independent set is an independent set which realizes the critical difference d and has maximum cardinality.



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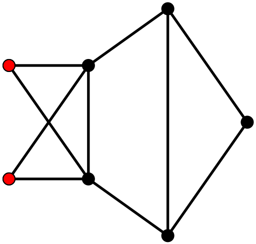

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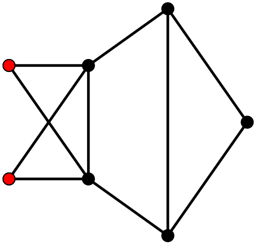

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I_c is a maximum cardinality critical independent set.

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4. for every maximum critical independent set J_c of G, $X = J_c \cup N(J_c)$.

• L., The Critical Independence Number and an Independence Decomposition, *European Journal of Combinatorics*, 2011.

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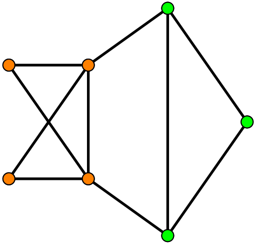

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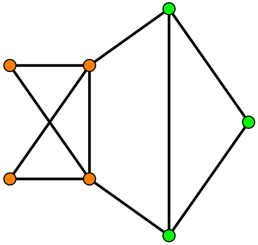

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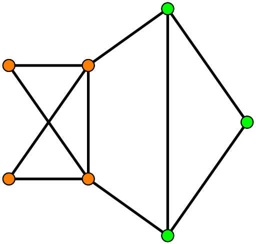

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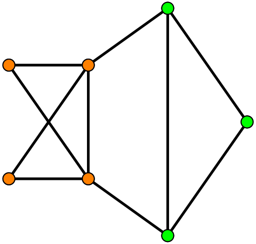

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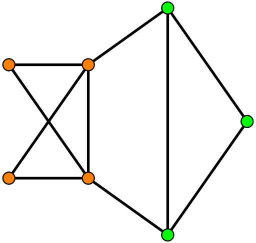

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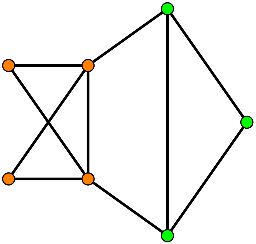

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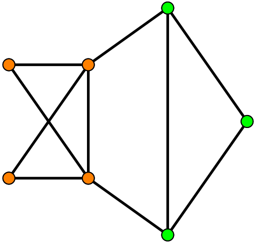

- $\alpha(G) = \alpha(G[X]) + \alpha(G[X^c]) = 3$.
- Every graph decomposes into a KE graph and a graph where every independent set I has more than $|I|$ neighbors.

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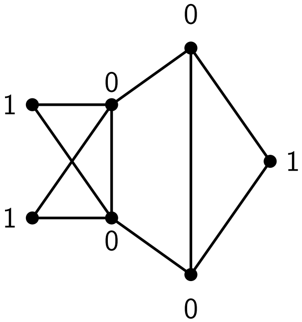

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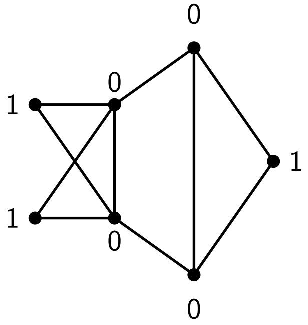

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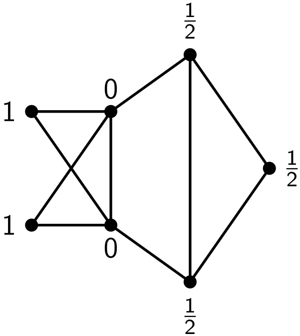

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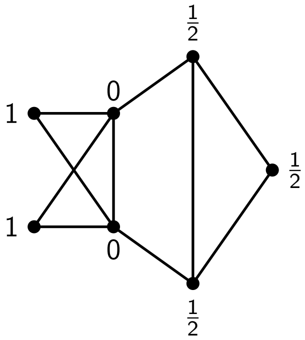
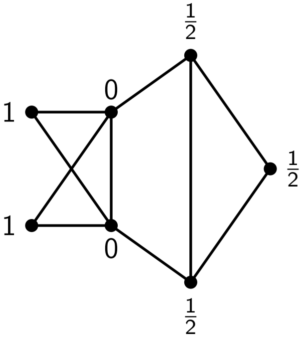


$$\alpha \le \max \sum w(v_i).$$

A Useful Result

Theorem

(Balinsky, 1965) There is an optimal solution to VPLP with weights $w(v_i) \in \{0, 1, \frac{1}{2}\}$.



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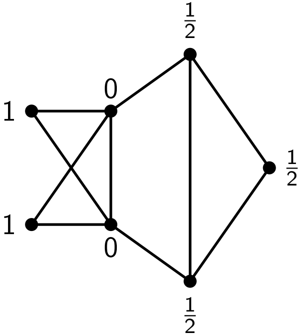

$$\max \sum w(v_i) = 3.5.$$

• G. L. Nemhauser and L. E. Trotter, "Properties of vertex packing and independence system polyhedra," in *Mathematical Programming*, 1974.

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Theorem

(Picard, Queyranne) If V_0, V_1, $V_{\frac{1}{2}}$ and V'_0, V'_1, $V'_{\frac{1}{2}}$ are optimal solutions with a maximum number of integral variables, then $V_0 \cup V_1 = V'_0 \cup V'_1$.

• J-C. Picard, M. Queyranne, "On the Integer-Valued Variables in the Linear Vertex Packing Problem", *Mathematical Programming*, 1977.

Facts

For a optimal solution $V_0, V_1, V_{\frac{1}{2}}$ of VPLP, and a critical independent set I_c, . . .

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2. $V_0 = N(V_1)$.

Facts

For a optimal solution $V_0, V_1, V_{\frac{1}{2}}$ of VPLP, and a critical independent set I_c, ...

1. $|V_0| + |V_1| + |V_{\frac{1}{2}}| = |I_c| + |N(I_c)| + |(I_c \cup N(I_c))^c|$.
2. $V_0 = N(V_1)$.
3. $|I_c| - |N(I_c)| \geq |V_1| - |V_0|$.

Optimal Solutions give Critical Independent Sets

Let $V_0, V_1, V_{\frac{1}{2}}$ be a feasible solution of VPLP with $N(V_1) = V_0$, and I_c be a critical independent set, ...

Theorem

$V_0, V_1, V_{\frac{1}{2}}$ is an optimal solution of VPLP if, and only if, V_1 is a critical independent set.

Picard-Queyranne Decomposition = Independence Decomposition

Theorem

$V_0, V_1, V_{\frac{1}{2}}$ is an optimal solution with a maximum number of integral variables if, and only if, V_1 is a maximum critical independent set.

Picard-Queyranne Decomposition = Independence Decomposition

Corollary

If $V_0, V_1, V_{\frac{1}{2}}$ *is an optimal solution with a maximum number of integral variables then* $V_0 \cup V_1 = X$*, from IDT.*

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(Picard, Queyranne) There are a unique maximal set of variables which are integral in optimal VPLP solutions.

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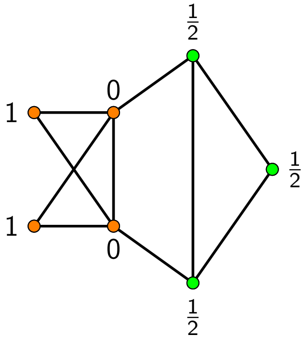
(Edmonds, L.) Picard-Queyranne Decomposition = Independence Decomposition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1
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Separable Independent Sets

An independent set I is separable if

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Note: critical independent sets are separable independent sets.

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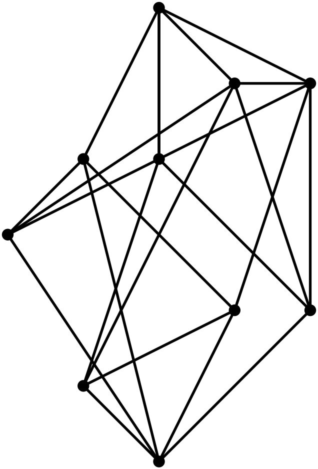

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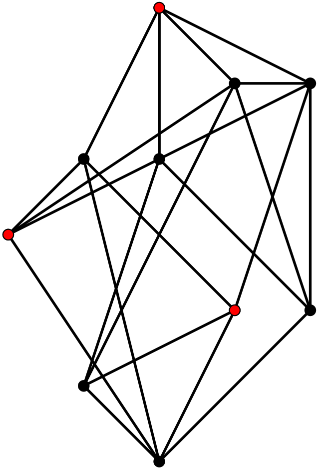

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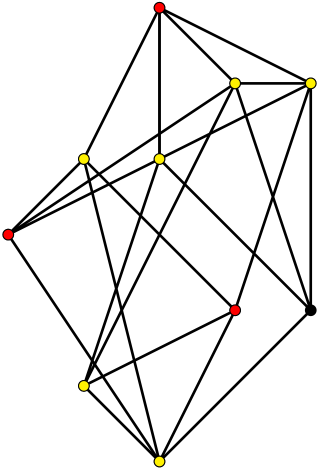

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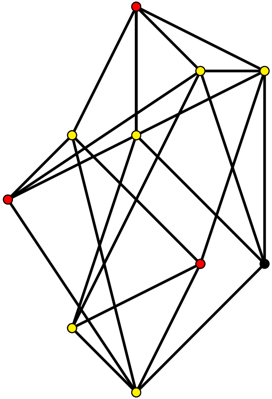

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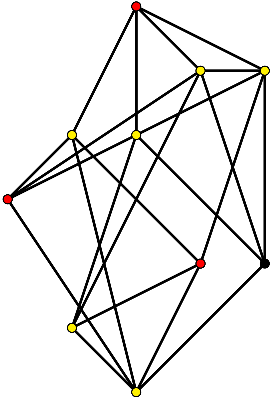

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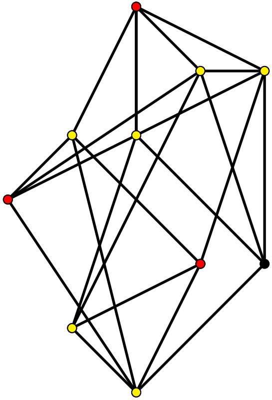

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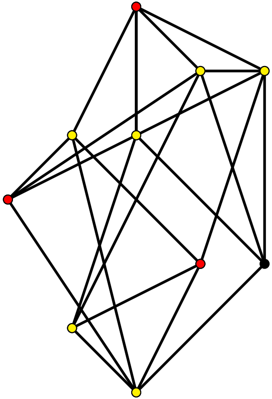

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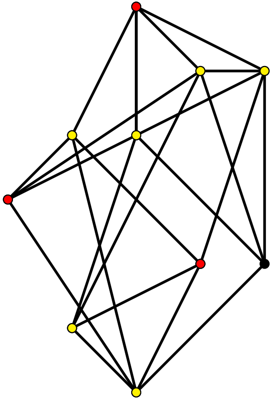
Separable Independent Sets



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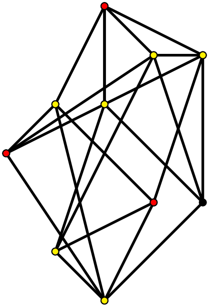
5. I is a separable independent set.

Problem 1



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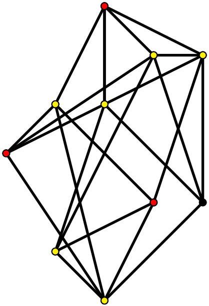

- How to Efficiently Identify Separable Independent Sets?

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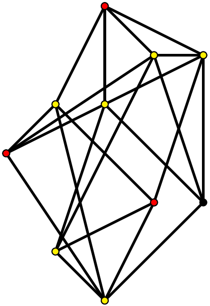

- How to Efficiently Identify Separable Independent Sets?
- What kinds of separable independent sets are there?

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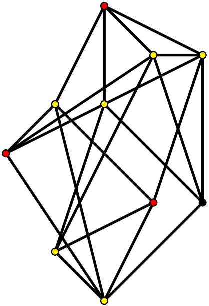
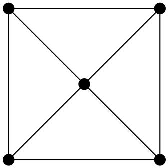


- How to Efficiently Identify Separable Independent Sets?
- What kinds of separable independent sets are there?
- Which kinds can be identified efficiently?

III. Efficient Computation of the Independence Number

When can independence number be computed efficiently?

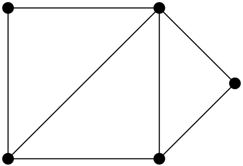
For claw-free graphs.



• G. Minty, On maximal independent sets of vertices in claw-free graphs, Journal of Combinatorial Theory. Series B, 28 (1980) 284–304.

When can independence number be computed efficiently?

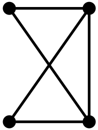
For perfect graphs.



• M. Chudnovsky, G. Cornuéjols, X. Liu, P. Seymour, K. Vušković, Recognizing Berge graphs, Combinatorica 25 (2005) 143–186.

When can independence number be computed efficiently?

For Bipartite and König-Egerváry graphs:



- König, Egerváry, 1931; Kuhn, 1955; Deming, Sterboul, 1979.

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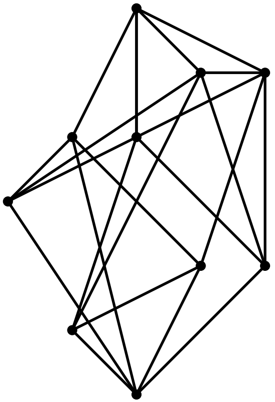
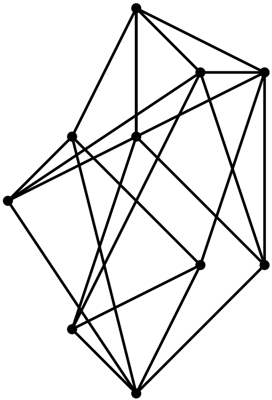


Problem 2



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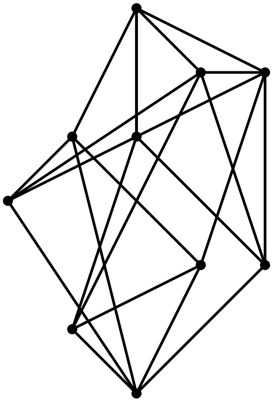

- Find New Forbidden Subgraph Characterizations

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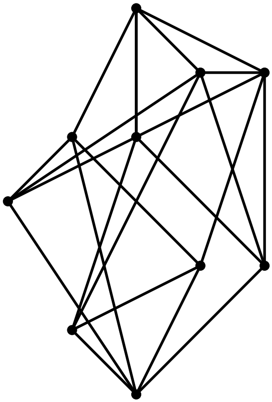


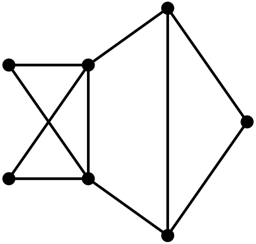
- Find New Forbidden Subgraph Characterizations
- Is it true that the independence number of P_5-free graphs can be computed efficiently?

When can independence number be computed efficiently?

When $l(G) = u(G)$, for an efficiently computable lower bound $l \leq \alpha$ and efficiently computable upper bound $\alpha \leq u$.

Residue Lower Bound

Given a graph G with degree sequence (d) the residue is the number of zeros at the result of the Havel-Hakimi process.



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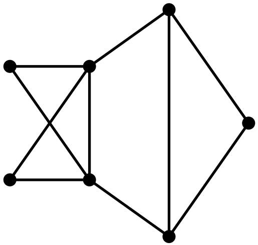

- $4, 4, 3, 3, 2, 2, 2.$

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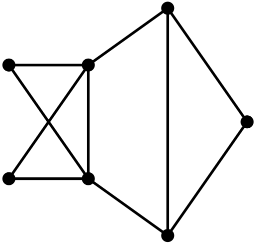

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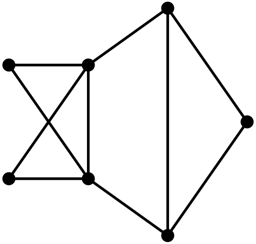

- $4, 4, 3, 3, 2, 2, 2.$
- $0, 0, 0.$
- $R = 3.$

$R \leq \alpha$ Graffiti, 1988; Favaron, Maheo, Sacle, 1991; Griggs, Kleitman, 1994.

Cvetkovíc Eigenvalues Bound

- Cvetkovic bound: $\alpha \leq$ min$\{\#$ of non-negative eigenvalues, $\#$ of non-positive eigenvalues$\}$

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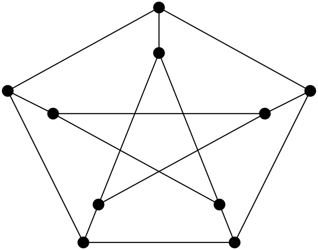

- Eigenvalues: 3, 1, 1, 1, 1, 1, -2, -2, -2, -2.

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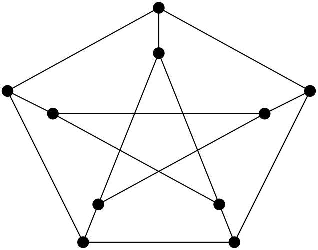
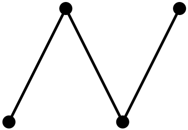


• Eigenvalues: 3, 1, 1, 1, 1, 1, -2, -2, -2, -2.

• $\alpha \leq 4$.

• D. Cvetković, M. Doob, and H. Sachs, Spectra of Graphs, 3rd ed., 1995.

When can independence number be computed efficiently?



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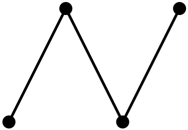

- $2 = Residue \leq \alpha \leq Cvetkovic = 2$

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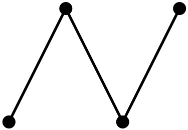

- $2 = \textit{Residue} \leq \alpha \leq \textit{Cvetkovic} = 2$
- Independence Number Theory implies $\alpha = 2$.

Residue is a good lower bound for α

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For connected graphs with minimum degree ≥ 3 and maximum degree $\leq n - 2$,

Residue is a good lower bound for α

For connected graphs with minimum degree ≥ 3 and maximum degree $\leq n-2$,

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For connected graphs with minimum degree ≥ 3 and maximum degree $\leq n-2$,

- Residue predicts α for 6 out of 8 graphs of order 6.
- Residue predicts α for 38 out of 88 graphs of order 7.

Residue is a good lower bound for α

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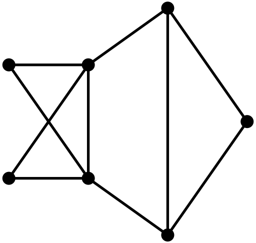
Problem 3



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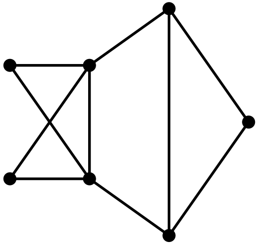

$$\alpha = \textit{residue} = 3$$

- Characterize the Graphs where $\alpha = \textit{residue}$

Lovász Theta Function

- The Lovász number of a graph G is:

$$\vartheta(G) = \max[1 - \frac{\lambda_1(A)}{\lambda_n(A)}]$$

over all real matrices A with $a_{ij} = 0$ if $v_i \sim v_j$ in G, with eigenvalues $\lambda_1(A) \geq \ldots \lambda_n(A)$

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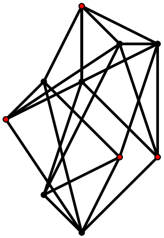

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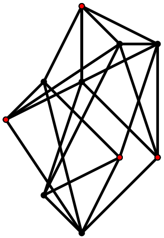

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- L. Lovász, On the Shannon capacity of a graph, *IEEE Transactions on Information Theory*, 1979.
- D. Knuth, The sandwich theorem, Electronic Journal of Combinatorics 1 (1994).

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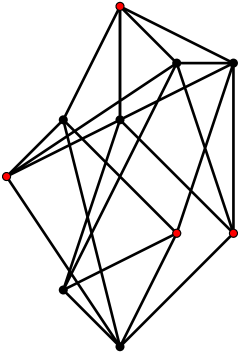
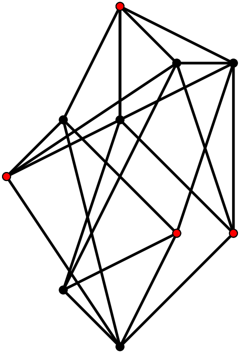


Problem 4



- Characterize Graphs Where $\alpha =$ Lovász Theta.

Efficiently Computable Bounds for the Independence Number

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1. Residue Lower Bound

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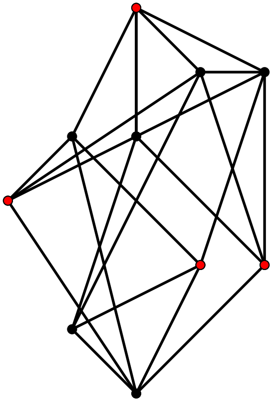
6. 50 Efficiently Computable Bounds are Known

Problem 5



- Find More Efficiently Computable Bounds for α.

When can independence number be computed efficiently?

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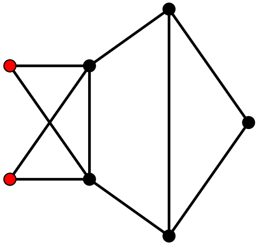
- When the graph has a simplicial vertex (that is, a vertex v, where $N[v]$ is complete. So $\alpha(G) = \alpha(G - v))$.

When can independence number be computed efficiently?



When G has a non-empty critical independent set, and α of $G[X^c]$ can be computed efficiently.

• L., A note on critical independence reductions, Bulletin of the ICA 51 (2007) 34–46.

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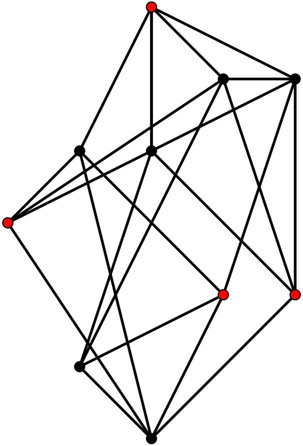
7. Has a magnet (Leveque, de Werra, 2012).

Problem 6



- Find new α-reductions.

IV. The Independence Number Project

Joint Work with Patrick Gaskill

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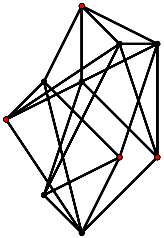

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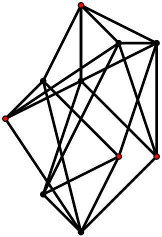

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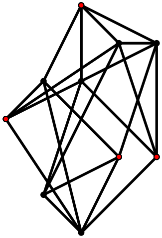

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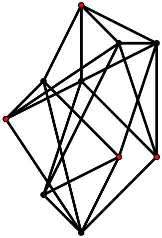

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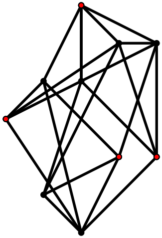

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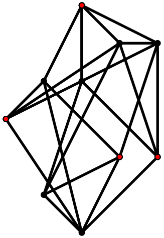

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- It demands new theory.

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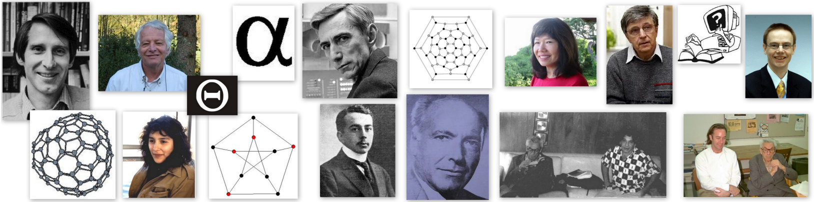
Thank You!



The Independence Number Project:

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