Proyecto Mate 4 P2 (Sarai Perez)

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Contents


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## PROBLEM 2
### POWER SERIES:
### Find the firs 6 nonzero terms in each of two linearly
### independent solutions of the form $\sum{c_nx^n} $, for
### the following differential equation:
### $ xy''+(sin x)y'+xy=0$
```

0.1 PROBLEM 2

0.1.1 POWER SERIES:

0.1.2 Find the firs 6 nonzero terms in each of two linearly

0.2 independent solutions of the form $\sum c_n x^n$, for

0.3 the following differential equation:

0.3.1 $xy'' + (sinx)y' + xy = 0$

```
#generating a sum with 12 terms.
reset()
n=12
a=list(var('a%d'%i)for i in range(n))
x=var('x')
y=function('y')(x)
var('a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11')
a0=0
a1=1
```

```
y(x)=a0+a1*x+a2*x^2+a3*x^3+a4*x^4+a5*x^5+a6*x^6+a7*x^7+a8*x^8+a9*x\
    ^9+a10*x^10+a11*x^11
show(y(x))
```

(a0, a1, a2, a3, a4, a5, a6, a7, a8, a9, a10, a11)

$$a_{11}x^{11} + a_{10}x^{10} + a_9x^9 + a_8x^8 + a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + x$$

```
#expanding the sine function using the McLaurin polynomial to use it\
    in the differential equation
sinX=taylor(sin(x),x,0,12)
show (sinX)
a0=0
a1=1
eq = expand (x*diff(y,x,2)+sinX*diff(y,x,1)+x*y(x)==0) #Substituting\
    the expansion in the differential equation given
show(eq)
```

$$-\frac{1}{39916800}x^{11} + \frac{1}{362880}x^9 - \frac{1}{5040}x^7 + \frac{1}{120}x^5 - \frac{1}{6}x^3 + x$$

$$x \mapsto -\frac{1}{3628800}a_{11}x^{21} - \frac{1}{3991680}a_{10}x^{20} + \frac{11}{362880}a_{11}x^{19} - \frac{1}{4435200}a_9x^{19} + \frac{1}{36288}a_{10}x^{18} - \frac{1}{4989600}a_8x^{18} - \frac{11}{5040}a_{11}x^{17} - \frac{1}{5702400}a_7x^{17} + \frac{1}{40320}a_9x^{17} - \frac{1}{504}a_{10}x^{16} - \frac{1}{6652800}a_6x^{16} + \frac{1}{45360}a_8x^{16} + \frac{11}{120}a_{11}x^{15} - \frac{1}{7983360}a_5x^{15} + \frac{1}{51840}a_7x^{15} - \frac{1}{560}a_9x^{15} + \frac{1}{12}a_{10}x^{14} - \frac{1}{9979200}a_4x^{14} + \frac{1}{60480}a_6x^{14} - \frac{1}{630}a_8x^{14} - \frac{11}{6}a_{11}x^{13} - \frac{1}{13305600}a_3x^{13} + \frac{1}{72576}a_5x^{13} - \frac{1}{720}a_7x^{13} + \frac{3}{40}a_9x^{13} - \frac{5}{3}a_{10}x^{12} + a_{11}x^{12} - \frac{1}{19958400}a_2x^{12} + \frac{1}{90720}a_4x^{12} - \frac{1}{840}a_6x^{12} + \frac{1}{15}a_8x^{12} + a_{10}x^{11} + 11\,a_{11}x^{11} + \frac{1}{120960}a_3x^{11} - \frac{1}{1008}a_5x^{11} + \frac{7}{120}a_7x^{11} - \frac{3}{2}a_9x^{11} + 10\,a_{10}x^{10} + 110\,a_{11}x^{10} + \frac{1}{181440}a_2x^{10} - \frac{1}{1260}a_4x^{10} + \frac{1}{20}a_6x^{10} - \frac{4}{3}a_8x^{10} + a_9x^{10} - \frac{1}{39916800}x^{11} + 90\,a_{10}x^9 - \frac{1}{1680}a_3x^9 + \frac{1}{24}a_5x^9 - \frac{7}{6}a_7x^9 + a_8x^9 + 9\,a_9x^9 - \frac{1}{2520}a_2x^8 + \frac{1}{30}a_4x^8 - a_6x^8 + a_7x^8 + 8\,a_8x^8 + 72\,a_9x^8 + \frac{1}{362880}x^9 + \frac{1}{40}a_3x^7 - \frac{5}{6}a_5x^7 + a_6x^7 + 7\,a_7x^7 + 56\,a_8x^7 + \frac{1}{60}a_2x^6 - \frac{2}{3}a_4x^6 + a_5x^6 + 6\,a_6x^6 + 42\,a_7x^6 - \frac{1}{5040}x^7 - \frac{1}{2}a_3x^5 + a_4x^5 + 5\,a_5x^5 + 30\,a_6x^5 - \frac{1}{3}a_2x^4 + a_3x^4 + 4\,a_4x^4 + 20\,a_5x^4 + \frac{1}{120}x^5 + a_2x^3 + 3\,a_3x^3 + 12\,a_4x^3 + 2\,a_2x^2 + 6\,a_3x^2 - \frac{1}{6}x^3 + 2\,a_2x + x^2 + x = 0$$

```
#grouping the coefficients of x and solving the system to find a2 we\
    have :
solve ([eq.lhs().coefficient(x,1)==0],a2)
show (solve([eq.lhs().coefficient(x,1)==0],a2))
```

[a2 == (-1/2)]

$$[a_2 = \left(-\frac{1}{2}\right)]$$

```
#performing this process recursively to find the other coefficients \
    of our series we have :
show (solve([eq.lhs().coefficient(x,2).substitute(a2=-1/2)==0],a3))
```

$[a_3 = 0]$

```
show (solve([eq.lhs().coefficient(x,3).substitute(a2=-1/2,a3=0)==0],\
    a4))
```

$[a_4 = \left(\frac{1}{18}\right)]$

```
show (solve([eq.lhs().coefficient(x,4).substitute(a2=-1/2,a3=0,a4\
    =1/18)==0],a5))
```

$[a_5 = \left(-\frac{7}{360}\right)]$

```
show (solve([eq.lhs().coefficient(x,5).substitute(a2=-1/2,a3=0,a4\
    =1/18,a5=-7/360)==0],a6))
```

$[a_6 = \left(\frac{1}{900}\right)]$

```
show (solve([eq.lhs().coefficient(x,6).substitute(a2=-1/2,a3=0,a4\
    =1/18,a5=-7/360,a6=1/900)==0],a7))
```

$[a_7 = \left(\frac{157}{113400}\right)]$

```
show (solve([eq.lhs().coefficient(x,7).substitute(a2=-1/2,a3=0,a4\
    =1/18,a5=-7/360,a6=1/900,a7=157/113400)==0],a8))
```

$[a_8 = \left(-\frac{19}{39690}\right)]$

```
show (solve([eq.lhs().coefficient(x,8).substitute(a2=-1/2,a3=0,a4\
    =1/18,a5=-7/360,a6=1/900,a7=157/113400,a8=-19/39690)==0],a9))
```

$[a_9 = \left(\frac{797}{38102400}\right)]$

```
show (solve([eq.lhs().coefficient(x,9).substitute(a2=-1/2,a3=0,a4\
    =1/18,a5=-7/360,a6=1/900,a7=157/113400,a8=-19/39690,a9\
    =797/38102400)==0],a10))
```

$[a_{10} = \left(\frac{923}{30618000}\right)]$

```
show (solve([eq.lhs().coefficient(x,10).substitute(a2=-1/2,a3=0,a4\
    =1/18,a5=-7/360,a6=1/900,a7=157/113400,a8=-19/39690,a9\
    =797/38102400,a10=923/30618000)==0],a11))
```

$[a_{11} = \left(-\frac{415519}{47151720000}\right)]$

```
#Now that we have calculated the coefficients we can calculate our \
    first solution
y1 (x)=0+1*x+(-1/2)*x^2+(0)*x^3+(1/18)*x^4+(-7/360)*x^5+(1/900)*x\
    ^6+(157/113400)*x^7+(-19/39690)*x^8+(797/38102400)*x\
```

```
    ^9+(923/30618000)*x^10+(-415519/47151720000)*x^11
show (y1(x))
```

$$-\frac{415519}{47151720000}x^{11}+\frac{923}{30618000}x^{10}+\frac{797}{38102400}x^{9}-\frac{19}{39690}x^{8}+\frac{157}{113400}x^{7}+\frac{1}{900}x^{6}-\frac{7}{360}x^{5}+\frac{1}{18}x^{4}-\frac{1}{2}x^{2}+x$$

```
#now we do the analogous process for our second linearly independent\
    solution
#generating a sum with 12 terms .
reset ()
n=12
a= list (var('a%d'%i) for i in range (n))
x=var('x')
y=function('y')(x)
var('a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11')
a0=1
a1=0
y(x)=a0+a1*x+a2*x^2+a3*x^3+a4*x^4+a5*x^5+a6*x^6+a7*x^7+a8*x^8+a9*x\
    ^9+a10*x^10+a11*x^11
show(y(x))
```

(a0, a1, a2, a3, a4, a5, a6, a7, a8, a9, a10, a11)

$$a_{11}x^{11}+a_{10}x^{10}+a_{9}x^{9}+a_{8}x^{8}+a_{7}x^{7}+a_{6}x^{6}+a_{5}x^{5}+a_{4}x^{4}+a_{3}x^{3}+a_{2}x^{2}+1$$

```
#expanding the sine function using the Mclaurin polynomial to use it\
    in the differential equation
senoX=taylor(sin(x),x,0,12)
show (senoX)
a0=1
a1=0
eq = expand (x*diff(y,x,2)+senoX*diff(y,x,1)+x*y(x)==0)
# Substituting the expansion in the differential equation given
show(eq)
```

$$-\frac{1}{39916800}x^{11}+\frac{1}{362880}x^{9}-\frac{1}{5040}x^{7}+\frac{1}{120}x^{5}-\frac{1}{6}x^{3}+x$$

$$x\mapsto -\frac{1}{3628800}a_{11}x^{21}-\frac{1}{3991680}a_{10}x^{20}+\frac{11}{362880}a_{11}x^{19}-\frac{1}{4435200}a_{9}x^{19}+\frac{1}{36288}a_{10}x^{18}-\frac{1}{4989600}a_{8}x^{18}-\frac{11}{5040}a_{11}x^{17}-\frac{1}{5702400}a_{7}x^{17}+\frac{1}{40320}a_{9}x^{17}-\frac{1}{504}a_{10}x^{16}-\frac{1}{6652800}a_{6}x^{16}+\frac{1}{45360}a_{8}x^{16}+\frac{11}{120}a_{11}x^{15}-\frac{1}{7983360}a_{5}x^{15}+\frac{1}{51840}a_{7}x^{15}-\frac{1}{560}a_{9}x^{15}+\frac{1}{12}a_{10}x^{14}-\frac{1}{9979200}a_{4}x^{14}+\frac{1}{60480}a_{6}x^{14}-\frac{1}{630}a_{8}x^{14}-\frac{11}{6}a_{11}x^{13}-\frac{1}{13305600}a_{3}x^{13}+\frac{1}{72576}a_{5}x^{13}-\frac{1}{720}a_{7}x^{13}+\frac{3}{40}a_{9}x^{13}-\frac{5}{3}a_{10}x^{12}+a_{11}x^{12}-\frac{1}{19958400}a_{2}x^{12}+\frac{1}{90720}a_{4}x^{12}-\frac{1}{840}a_{6}x^{12}+\frac{1}{15}a_{8}x^{12}+a_{10}x^{11}+11\,a_{11}x^{11}+\frac{1}{120960}a_{3}x^{11}-\frac{1}{1008}a_{5}x^{11}+\frac{7}{120}a_{7}x^{11}-\frac{3}{2}a_{9}x^{11}+10\,a_{10}x^{10}+110\,a_{11}x^{10}+\frac{1}{181440}a_{2}x^{10}-\frac{1}{1260}a_{4}x^{10}+\frac{1}{20}a_{6}x^{10}-\frac{4}{3}a_{8}x^{10}+a_{9}x^{10}+90\,a_{10}x^{9}-\frac{1}{1680}a_{3}x^{9}+\frac{1}{24}a_{5}x^{9}-\frac{7}{6}a_{7}x^{9}+a_{8}x^{9}+9\,a_{9}x^{9}-\frac{1}{2520}a_{2}x^{8}+\frac{1}{30}a_{4}x^{8}-a_{6}x^{8}+a_{7}x^{8}+8\,a_{8}x^{8}+72\,a_{9}x^{8}+\frac{1}{40}a_{3}x^{7}-\frac{5}{6}a_{5}x^{7}+a_{6}x^{7}+7\,a_{7}x^{7}+$$

$56\,a_8x^7 + \frac{1}{60}\,a_2x^6 - \frac{2}{3}\,a_4x^6 + a_5x^6 + 6\,a_6x^6 + 42\,a_7x^6 - \frac{1}{2}\,a_3x^5 + a_4x^5 + 5\,a_5x^5 + 30\,a_6x^5 - \frac{1}{3}\,a_2x^4 + a_3x^4 + 4\,a_4x^4 + 20\,a_5x^4 + a_2x^3 + 3\,a_3x^3 + 12\,a_4x^3 + 2\,a_2x^2 + 6\,a_3x^2 + 2\,a_2x + x = 0$

```
solve([eq.lhs().coefficient(x,1)==0],a2)
show(solve([eq.lhs().coefficient(x,1)==0],a2))
```

[a2 == (-1/2)]

$[a_2 = \left(-\frac{1}{2}\right)]$

```
show(solve([eq.lhs().coefficient(x,2).substitute(a2=-1/2)==0],a3))
```

$[a_3 = \left(\frac{1}{6}\right)]$

```
show(solve([eq.lhs().coefficient(x,3).substitute(a2=-1/2,a3=1/6)\
    ==0],a4))
```

$[a_4 = 0]$

```
show(solve([eq.lhs().coefficient(x,4).substitute(a2=-1/2,a3=1/6,a4\
    =0)==0],a5))
```

$[a_5 = \left(-\frac{1}{60}\right)]$

```
show(solve([eq.lhs().coefficient(x,5).substitute(a2=-1/2,a3=1/6,a4\
    =0,a5=-1/60)==0],a6))
```

$[a_6 = \left(\frac{1}{180}\right)]$

```
show(solve([eq.lhs().coefficient(x,6).substitute(a2=-1/2,a3=1/6,a4\
    =0,a5=-1/60,a6=1/180)==0],a7))
```

$[a_7 = \left(-\frac{1}{5040}\right)]$

```
show(solve([eq.lhs().coefficient(x,7).substitute(a2=-1/2,a3=1/6,a4\
    =0,a5=-1/60,a6=1/180,a7=-1/5040)==0],a8))
```

$[a_8 = \left(-\frac{1}{2520}\right)]$

```
show(solve([eq.lhs().coefficient(x,8).substitute(a2=-1/2,a3 =1/6,a4\
    =0,a5=-1/60,a6=1/180,a7=-1/5040,a8=-1/2520)==0],a9))
```

$[a_9 = \left(\frac{11}{90720}\right)]$

```
show(solve([eq.lhs().coefficient(x,9).substitute(a2=-1/2,a3 =1/6,a4\
    =0,a5=-1/60,a6=1/180,a7=-1/5040,a8=-1/2520,a9=11/90720)==0],a10))
```

$[a_{10} = \left(-\frac{1}{680400}\right)]$

```
show(solve([eq.lhs().coefficient(x,10).substitute(a2=-1/2,a3=1/6,a4\
    =0,a5=-1/60,a6=1/180,a7=-1/5040,a8=-1/2520,a9=11/90720,a10\
    =-1/680400)==0],a11))
```

$$[a_{11} = \left(-\frac{4957}{598752000}\right)]$$

```
y2(x)=1+0*x+(-1/2)*x^2+(1/6)*x^3+(0)*x^4+(-1/60)*x^5+(1/180)*x\
    ^6+(-1/5040)*x^7+(-1/2520)*x^8+(11/90720)*x^9+(-1/680400)*x\
    ^10+(-4957/598752000)*x^11
show(y2(x))
```

$$-\frac{4957}{598752000}x^{11}-\frac{1}{680400}x^{10}+\frac{11}{90720}x^{9}-\frac{1}{2520}x^{8}-\frac{1}{5040}x^{7}+\frac{1}{180}x^{6}-\frac{1}{60}x^{5}+\frac{1}{6}x^{3}-\frac{1}{2}x^{2}+1$$