Section 1.2

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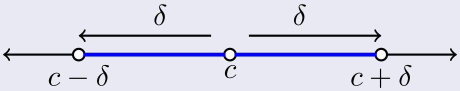
Definition (Punctured δ-interval)

Let there exist a real numbers c and $\delta > 0$. Then the **punctured interval** is the open interval $(c-\delta, c) \cup (c, c+\delta)$.



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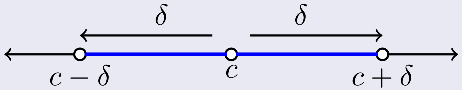

Basic Idea

- The value c is an x-value that you wish to get really close to and δ is how close you want to be to c.

Definition (ε-δ Definition of a Limit)

If $f(x)$ is a function where $\lim_{x \to c} f(x) = L$, then

$$\forall_{\varepsilon > 0}\, \exists_{\delta > 0} \text{ if } x \in (c - \delta, c) \cup (c, c + \delta) \Rightarrow L \in (L - \varepsilon, L + \varepsilon).$$

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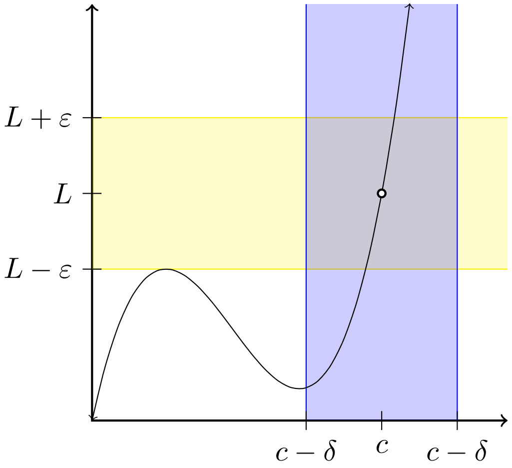
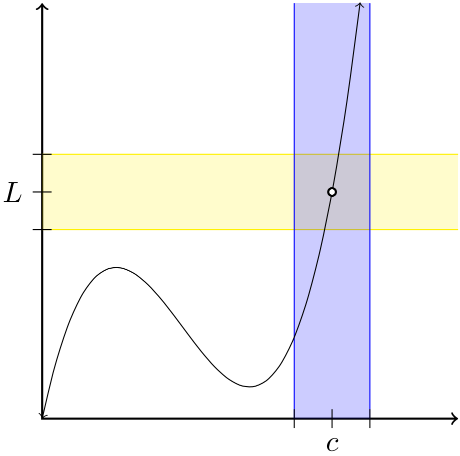
Let's look at an illustration of this definition.

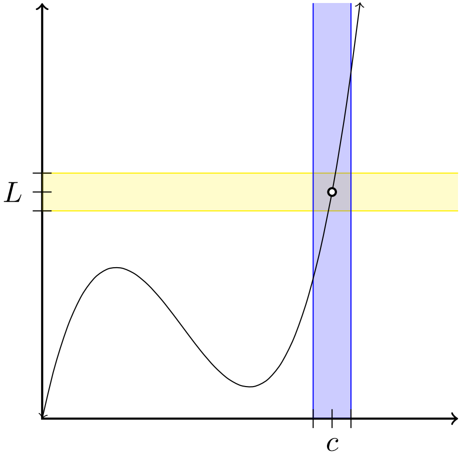
$\varepsilon = 2$.



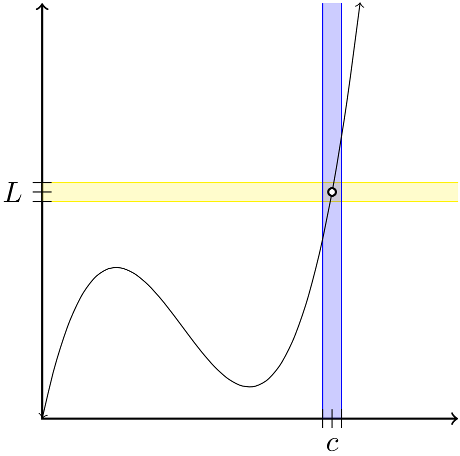
$\varepsilon = 1.$



$\varepsilon = 0.5.$



$\varepsilon = 0.25.$



Basic Idea

No matter what value of ε I choose, I should be able to find a δ that will enclose the limit value inside of some rectangle.

Example (Finding a δ for a given ε – Example 1)

Let $f(x) = x^3$. Given $\varepsilon = 0.25$, find a δ so that

$$x \in (2-\delta, 2) \cup (2, 2+\delta) \Rightarrow x^3 \in (8-\varepsilon, 8+\varepsilon).$$

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$$x \in (2-\delta, 2) \cup (2, 2+\delta) \Rightarrow x^3 \in (8-\varepsilon, 8+\varepsilon).$$

Since $\varepsilon = 0.25$, we know that $x^3 \in (7.75, 8.25)$. This means we need to find what the lower and upper x-values are. This happens when $x^3 = 7.75$ and $x^3 = 8.25$.

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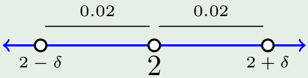
The lower x-value is 1.98 and the upper x-value is 2.02. Since the c value is 2, this means δ must be 0.02. Below is a picture of this punctured δ-interval.



Example (Finding a δ for a given ε – Example 2)

Let $f(x) = \sqrt{x}$. Given $\varepsilon = 0.10$, find a δ so that

$$x \in (4-\delta, 4) \cup (4, 4+\delta) \Rightarrow \sqrt{x} \in (2-\varepsilon, 2+\varepsilon).$$

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Since $\varepsilon = 0.10$, we know that $\sqrt{x} \in (1.9, 2.1)$. This means we need to find what the lower and upper x-values are. This happens when $\sqrt{x} = 1.9$ and $\sqrt{x} = 2.1$.

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The lower x-value is 3.61 and the upper x-value is 4.41. Here we have a bit of an issue. The distance between 4 and the end points of the interval are not the same. This means we have two potential δ-values to choose from. The first is 0.39 and the second is 0.41. Which do we pick?

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We need to pick the smaller of the two in order to ensure the interval isn't too wide. So, we pick $\delta = 0.39$.