The Charge to Mass Ratio of an Electron
Physics 211

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Abstract

In this lab, we used a Helmholtz coil in order to observe and study the effect of a magnetic field on the path of a charged particle, more specifically, the path of an electron beam. We demonstrated the theoretically predicted relationship between the magnetic field strength and the curvature of the path of an electron. We did this by experimentally determining the ratio of an electron's charge to its mass (written e/m). We performed two trials in which we changed the magnetic field strength and the acceleration voltage of the electrons while measuring the diameter of the electron's circular path. We found e/m to be $\frac{e}{m} \pm \delta_{e/m} = (174 \pm 7) \times 10^9 \ \frac{\mathrm{C}}{\mathrm{kg}}$ and $\frac{e}{m} \pm \delta_{e/m} = (170 \pm 3) \times 10^9 \ \frac{\mathrm{C}}{\mathrm{kg}}$, the former of which agrees within uncertainty to the accepted value of $e/m = 1.759 \times 10^{11} \ \frac{\mathrm{C}}{\mathrm{kg}}$.

Contents

1 Introduction

The purpose of this experiment was to study the behavior of charged particles moving in magnetic fields. We did so by examining the path of electrons traveling in a magnetic field and determining the ratio of the electron charge, e to the electron mass, m in order to demonstrate the theory behind this phenomenon. We will call the ratio e/m the charge-to-mass ratio.

This experiment is connected to import discoveries in the field: Nobel Prize was awarded to J.J. Thompson for measuring e/m [1].

2 Theory

We know that while static magnetic fields do not exert forces on stationary charged particles, they can sometimes, however, exert a force on a charged particle that is moving through that field. The force that a magnetic field $\vec{B}$ exerts on a moving charged particle is given by

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \tag{1}$$

where the particle has charge q and velocity $\vec{v}$. From this equation we can deduce that since the force is perpendicular to the particle's velocity, the magnetic field does no work on the particle, implying that its kinetic energy, and thus speed, is not changed by the field.

Only the direction of motion is affected by the magnetic field. In the case where $\vec{v}$ is perpendicular to $\vec{B}$, the force will always be perpendicular to $\vec{v}$, so $\vec{F}_m$ will be a constant centripetal force causing the particle to move in a circular path [2]. When such a charged particle happens to be an electron, we will have that

$$|\vec{F}_m| = e|\vec{v}||\vec{B}| \tag{2}$$

is constant. From Newton's second Law, we can write

$$e|\vec{v}||\vec{B}| = |\vec{F}_m| = m\frac{|\vec{v}|^2}{r} \tag{3}$$

which (by dividing through by $|\vec{v}||\vec{B}|m$) implies that

$$\frac{e}{m} = \frac{|\vec{v}|}{r|\vec{B}|} \tag{4}$$

where r is the radius of the circular path previously discussed [1]. We can relate the total electrostatic potential energy of an electron V_e, to the potential ϕ at a point relative to infinity, by [2]

$$e\phi = V_e. \tag{5}$$

This means that in an electron gun, the potential energy gained by an electron moving from one plate (negatively charged) to the other (positively charged) is

$$\Delta V_e = e\phi_f - e\phi_i = e(\phi_f - \phi_i) = eV \tag{6}$$

where V is the (positive) potential difference across the plates. Conservation of energy requires that the negative change in potential energy is equal to the change in kinetic energy, ΔK:

$$-eV = -\Delta V_e = \Delta K = \frac{1}{2}m|\vec{v}|^2. \tag{7}$$

Note: e is a negative constant but for simplicity of calculations and because of the unruly negative sign in Eq. 7, we will let $-eV = eV$, so e/m is positive.

Solving Eq. 7 for $\vec{v}$ yields

$$\vec{v} = \sqrt{\frac{2eV}{m}} \tag{8}$$

and by substituting Eq. 8 into Eq. 4, we get that [2]

$$\frac{e}{m} = \frac{2V}{r^2|\vec{B}|^2}. \tag{9}$$

We also used an apparatus (discussed later) that produced a constant magnetic field that is proportional to a controlled current I. This relation is given by [1]

$$\left|\vec{B}\right| = (7.79 \times 10^{-4} \text{ T/A})I. \tag{10}$$

3 Experimental Procedure

3.1 Experimental Apparatus

We used a Helmholtz coil, which is a set of two circular coils of wire connected in series and mounted parallel to one another. When carrying current, these coils produce a constant magnetic field in the region between the coils and in the x direction (see Figure 1).

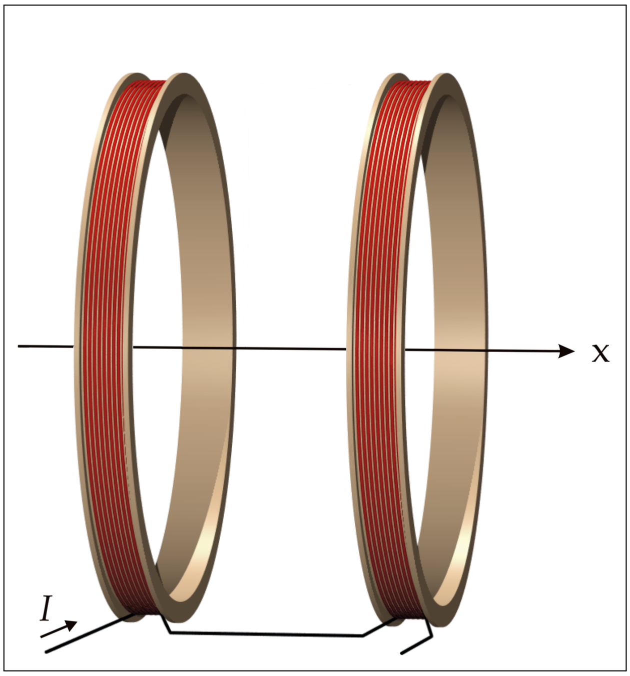

Figure 1: Helmholtz Coil [4]

Between the coils is a discharge tub which contains low pressure helium and an electron gun on one end that injects electrons into the tube that excite the helium gas and make an aqua-colored illuminated path. There is a glass ruler that measures the distance from the electron gun in the imaginary plane centered between the two coils. The electrons are then attracted to an anode so they can safely leave the tube after being measured (see Figure 2).

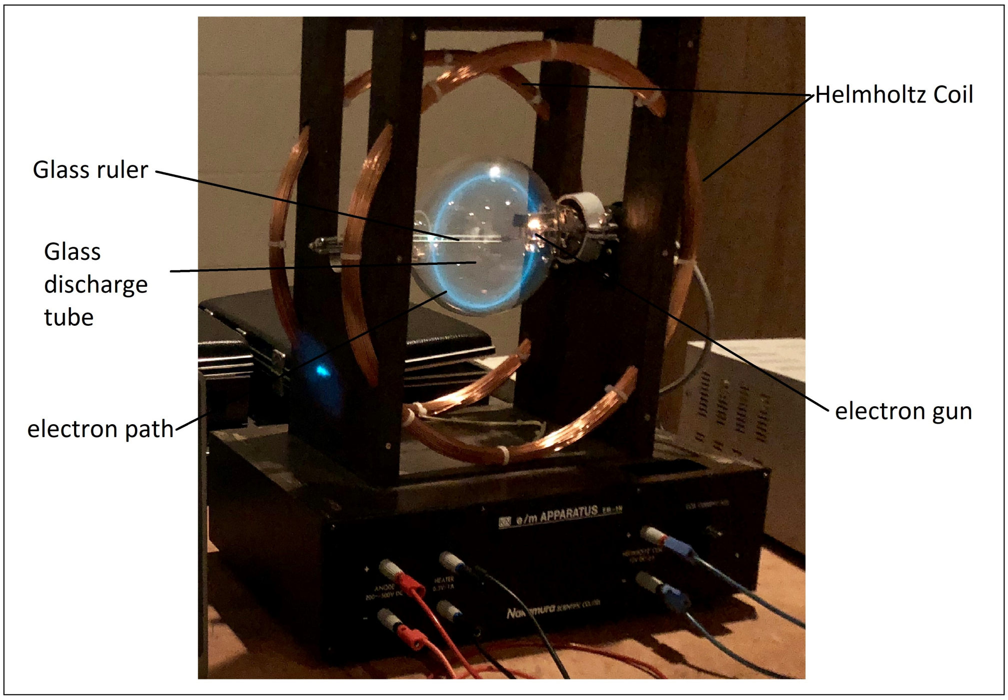

Figure 2: Nakamura KN e/m Apparatus EM-1N

The coil apparatus is connected to two power supplies. The B-power supply produces the potential difference (acceleration voltage) in the electron gun that affects the velocity of the electrons. The coil power supply provides the current in the coils that produces the magnetic field in the apparatus (see Figure 3).

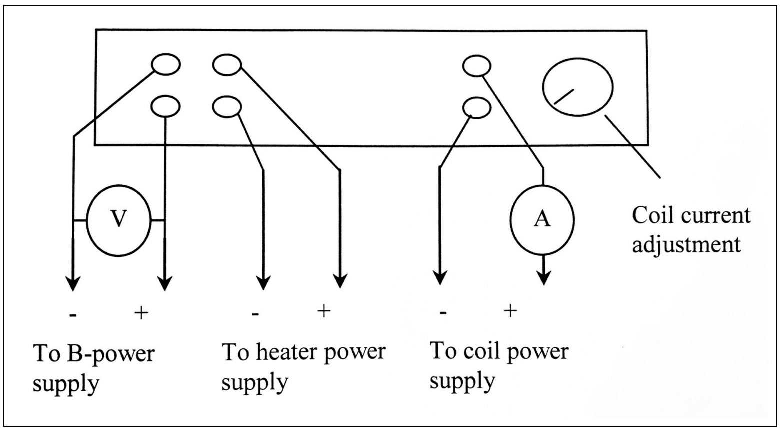

Figure 3: Front Panel of e/m Apparatus [1]

The voltmeter and Ammeter shown in Figure 3 are used to measure the acceleration voltage and the current through the coils [1].

3.2 Procedure

We performed two trials on the e/m apparatus. In trial 1, we held the acceleration voltage to be constant and varied the magnetic field created by changing the current through the coils. We decided to use an acceleration voltage of 351 Volts to reduce error.

By contrast, in trial 2 we maintained the magnetic field while varying the acceleration voltage. In both trials we measured the diameter of the electron's circular path in the gas tube. We decided to use a constant current of 1530 mA through the coils to reduce error.

4 Data and Analysis

4.1 Measured Quantities

The following table shows the measured data from trial 1 in the first three columns and direct calculations on that measured data in the other four columns. Column 4 is calculated applying Eq. 10 to column 2.

Acceleration Voltage V (Volts)	Coil Current I (mA)	Diameter of path d (cm)	Magnetic field B (T)	Inverse square of Magnetic Field $1/B^2$ (T^{-2})	Radius of path $r \equiv d/2$ (m)	square of radius r^2 (m^2)
351	1962	7.5	0.001528398	428082.11	0.0375	0.001406
351	1882	8	0.001466078	465249.43	0.04	0.001600
351	1795	8.5	0.001398305	511441.75	0.0425	0.001806
351	1712	9	0.001333648	562234.59	0.045	0.002025
351	1637	9.5	0.001275223	614932.90	0.0475	0.002256
351	1566	10	0.001219914	671957.13	0.05	0.002500
351	1496	10.5	0.001165384	736312.03	0.0525	0.002756
351	1397	11	0.001088263	844368.98	0.055	0.003025

Table 1: Recorded and calculated data from Trial 1.

The following table shows the measured data from the trial 2 in the first three columns and direct calculations on that measured data in the other three columns. Column 4 is calculated applying Eq. 10 to column 2.

Acceleration Voltage V (Volts)	Coil Current I (mA)	Diameter of path d (cm)	Magnetic field B (T)	Radius of path $r = d/2$ (m)	square of radius r^2 (m^2)
101	1530	5	0.00119187	0.025	0.0006250
132	1530	5.5	0.00119187	0.0275	0.0007563
150	1530	6	0.00119187	0.03	0.0009000
162	1530	6.5	0.00119187	0.0325	0.0010563
180	1530	7	0.00119187	0.035	0.0012250
201	1530	7.5	0.00119187	0.0375	0.0014063
228	1530	8	0.00119187	0.04	0.0016000
249	1530	8.5	0.00119187	0.0425	0.0018063
279	1530	9	0.00119187	0.045	0.0020250
306	1530	9.5	0.00119187	0.0475	0.0022563
333	1530	10	0.00119187	0.05	0.0025000
372	1530	10.5	0.00119187	0.0525	0.0027563

Table 2: Recorded and calculated data from Trial 2.

4.2 Error Propagation

For our measured quantities, we decided to use 1/2 of the smallest possible measurement as our error for that measurement. Using formulas outlined and thoroughly explained in [3], we can express the error in the magnetic field, r^2, and $1/B^2$ as

$$\delta_B = (7.79 \times 10^{-4}\ \mathrm{T/A})\delta_I \tag{11}$$

$$\delta_{r^2} = \frac{1}{2} d\delta_d \tag{12}$$

$$\delta_{1/B^2} = 2B^{-3}\delta_B. \tag{13}$$

In Table 3, columns 1,2 and 4 are given by the half-measurement described previously. Column 3 is given by Eq. 11 and columns 5 and 6 are given by Eq. 12 and Eq. 13, respectively.

Error in Acceleration Voltage δ_V (Volts)	Error in Current δ_I (A)	Error in Magnetic Field δ_B (T)	Error in diameter δ_d (cm)	Error in r^2 δ_{r^2} (m^2)	Error in $1/B^2$ δ_{1/B^2} (T^{-2})
0.5	0.0005	3.895×10^{-7}	0.25	0.00009375	218.1866004
0.5	0.0005	3.895×10^{-7}	0.25	0.0001	247.2101097
0.5	0.0005	3.895×10^{-7}	0.25	0.00010625	284.9257669
0.5	0.0005	3.895×10^{-7}	0.25	0.0001125	328.4080572
0.5	0.0005	3.895×10^{-7}	0.25	0.00011875	375.6462402
0.5	0.0005	3.895×10^{-7}	0.25	0.000125	429.0914011
0.5	0.0005	3.895×10^{-7}	0.25	0.00013125	492.1871875
0.5	0.0005	3.895×10^{-7}	0.25	0.0001375	604.4158802

Table 3: Trial 1 error propagation.

In Table 4, columns 1,2 and 4 are given by the half-measurement described previously. Column 3 is given by Eq. 11 and columns 5 is given by Eq. 12.

Error in Acceleration Voltage δ_V (Volts)	Error in Current δ_I (A)	Error in Magnetic Field δ_B (T)	Error in diameter δ_d (cm)	Error in r^2 δ_{r^2} (m^2)
0.5	0.5	0.0003895	0.25	0.0000625
0.5	0.5	0.0003895	0.25	0.00006875
0.5	0.5	0.0003895	0.25	0.000075
0.5	0.5	0.0003895	0.25	0.00008125
0.5	0.5	0.0003895	0.25	0.0000875
0.5	0.5	0.0003895	0.25	0.00009375
0.5	0.5	0.0003895	0.25	0.0001
0.5	0.5	0.0003895	0.25	0.00010625
0.5	0.5	0.0003895	0.25	0.0001125
0.5	0.5	0.0003895	0.25	0.00011875
0.5	0.5	0.0003895	0.25	0.000125
0.5	0.5	0.0003895	0.25	0.00013125

Table 4: Trial 2 error propagation.

4.3 Graphical Analysis for Trial 1

We can rearrange Eq. 9 as

$$\frac{1}{B^2} = \left(\frac{e}{m}\frac{1}{2V}\right) r^2 \tag{14}$$

so we can relate the slope of the line of best-fit for a plot of column 5 vs. column 7 from Table 1 to the quantity e/m (see Figure 4).

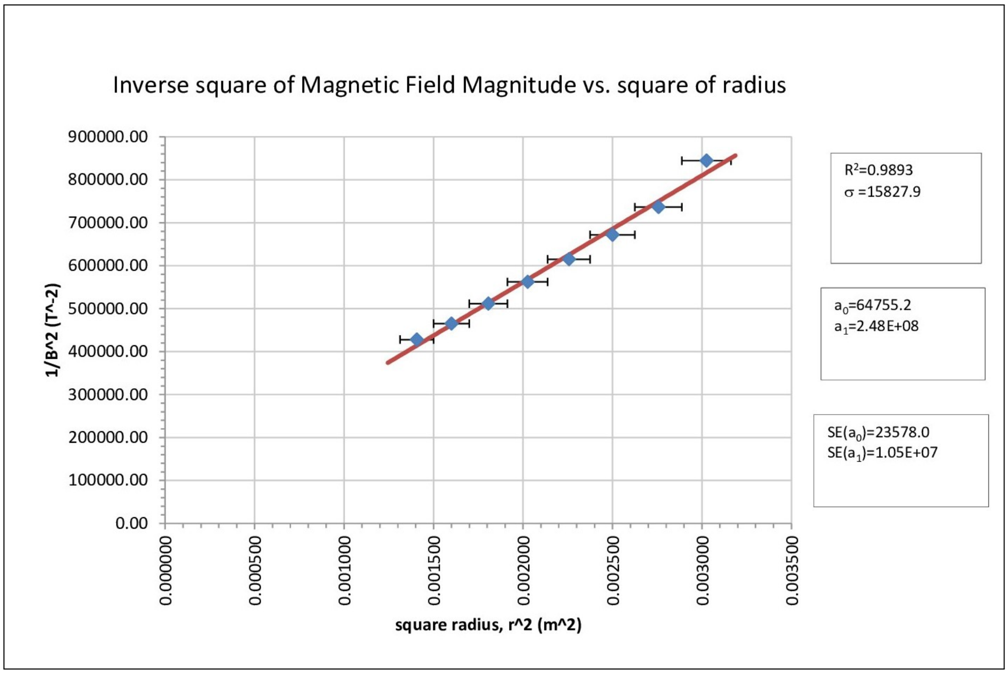

Figure 4: Plot and line of best-fit from columns 5 and 7 in Table 1

Error bars for Figure 4 come from columns 3 and 5 from Table 1.

From Eq. 19 we can see that the slope of this best-fit line is

$$a_1 = \frac{e}{m}\frac{1}{2V} \tag{15}$$

which means that

$$\frac{e}{m} = a_1(2V) = \left(2.48 \times 10^8 \ \frac{1}{\mathrm{T}^2\mathrm{m}^2}\right)(2)(351\mathrm{V}) = 1.74 \times 10^{11} \ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{16}$$

Using formulas outlined and thoroughly explained in [3], we can see that the error in e/m is given by

$$\delta_{e/m} = (2V)\delta_{a_1} = (2V)\mathrm{SE}(a_1) = (2)(351\ \mathrm{V})\left(1.05 \times 10^7 \ \frac{1}{\mathrm{T}^2\mathrm{m}^2}\right) = 7.371 \times 10^9 \ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{17}$$

We can conclude from trial 1 that our experimentally determined value of e/m along with its uncertainty is

$$\frac{e}{m} \pm \delta_{e/m} = (174 \pm 7) \times 10^9 \ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{18}$$

4.4 Graphical Analysis for Trial 2

We can rearrange Eq. 9 as

$$V = \left(\frac{e}{m}\frac{B^2}{2}\right) r^2 \tag{19}$$

so we can relate the slope of the line of best-fit for a plot of column 1 vs. column 6 from Table 2 to the quantity e/m (see Figure 5).

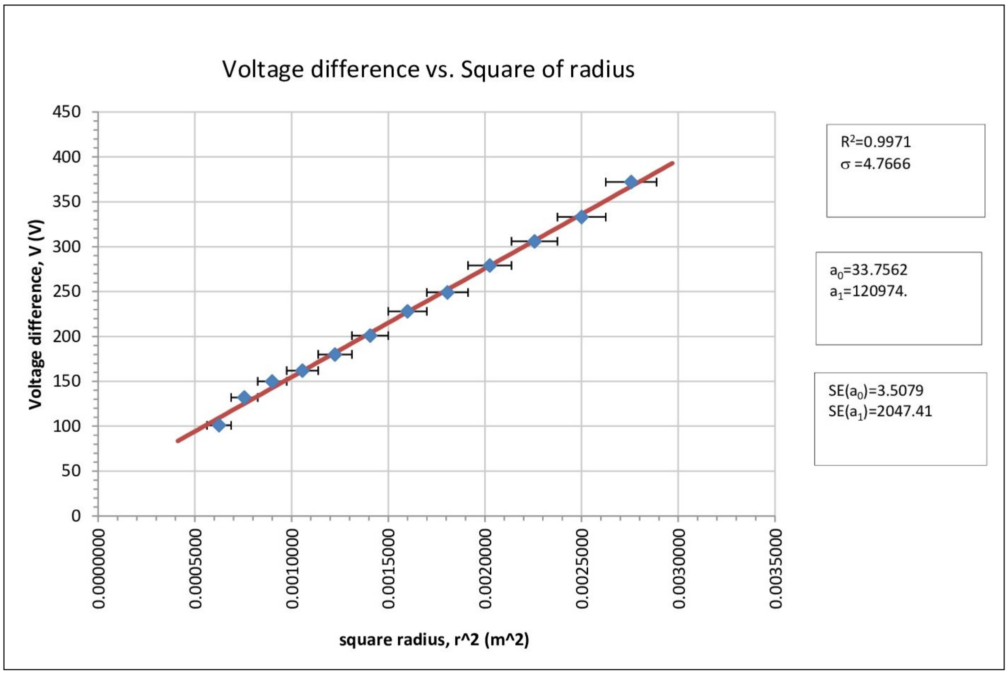

Figure 5: Plot and line of best-fit from column 1 vs. column 6 in Table 1

Error bars for Figure 4 come from columns 1 and 5 from Table 2.

From Eq. 19 we can see that the slope of this best-fit line is

$$a_1 = \frac{e}{m}\frac{B^2}{2} \tag{20}$$

which means that

$$\frac{e}{m} = \frac{2}{B^2}a_1 = \left(\frac{2}{(0.00119187\ \mathrm{T})^2}\right)\left(120974\ \frac{\mathrm{V}}{\mathrm{m}^2}\right) = 1.703194553\times 10^{11}\ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{21}$$

Using formulas outlined and thoroughly explained in [3], we can see that the error in e/m is given by

$$\delta_{e/m} = \frac{2}{B^2}\delta_{a_1} = \frac{2}{B^2}\mathrm{SE}(a_1) = \left(\frac{2}{(0.00119187\ \mathrm{T})^2}\right)\left(2047.41\ \frac{\mathrm{V}}{\mathrm{m}^2}\right) = 2.88\times 10^{9}\ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{22}$$

We can conclude from trial 2 that our experimentally determined value of e/m along with its uncertainty is

$$\frac{e}{m} \pm \delta_{e/m} = (170 \pm 3)\times 10^{9}\ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{23}$$

5 Results and Conclusion

In this lab, we determined the charge to mass ratio of an electron by studying the effect that changing the magnetic field created by running current through a Helmholtz coil and the acceleration voltage of the electron gun had on the path of an electron through a gas chamber that illuminates the electron's path. In trial 1, where we varied the magnetic field and kept the acceleration voltage constant, we found this ratio to be

$$\frac{e}{m} \pm \delta_{e/m} = (174 \pm 7) \times 10^{9} \ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{24}$$

In trial 2, where we changed the acceleration voltage and kept the magnetic field constant, we found this ratio to be

$$\frac{e}{m} \pm \delta_{e/m} = (170 \pm 3) \times 10^{9} \ \frac{\mathrm{C}}{\mathrm{kg}}. \tag{25}$$

The accepted value of e/m is $1.759 \times 10^{11} \ \frac{\mathrm{C}}{\mathrm{kg}}$ which is in agreement with out results from trial 1 but only close to our results from trial 2.

Some sources of random error are the inability to properly change the electron's path to lie directly on one of the half-centimeter markings (the only markings) on the ruler. Also, the source for the acceleration voltage did not keep at a constant value. These small fluctuations in voltage affect our results.

As long as the electron path falls on the ruler, the misalignment of the electron gun to exactly perpendicular to the magnetic field will not contribute systematic error. Since the magnetic force is only affecting the velocity component that is perpendicular to the field, at a given distance from the electron gun, the path will only be moved perpendicular to the ruler. As long as the path goes through the ruler, even if the path does not go through the center, the distance read on the ruler will be read as if it does.

Some resistance in the wire and apparatus between source readings and equipment can account for some systematic error. Since the wire and device might not be ohmic, it is difficult to analyze exactly how this effects the results but we know that this will change the voltage believed to be accelerating the electron.

References

[1] Physics 211: Intermediate Physics Labratory Manual, Gettysburg College Lab Handout (Unpublished).

[2] Thomas A. Moore, Six Ideas That Shaped Physics. Unit E, Electric and Magnetic Fields are Unified, Ed. 3. (New York, NY, McGraw-Hill Education, 2017)

[3] John R. Taylor, An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements. (California, University Science Books Sausalito, 1997)

[4] Ansgar Hellwig, Helmholtz coils, PNG Document, `https://commons.wikimedia.org/wiki/File:Helmholtz_coils.png`

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