Section 0.5–1.1

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Definition (The "For All" and "There Exists" Quantifiers)

Let P be some property that depends on a value of x.

(a) **For all** x**, property** P means that property P is true for all possible values of x.
The symbol $\forall$ is used to mean "for all".

(b) **There exists** x **such that property** P means that there is at least one value of x that makes property P true.
The symbol $\exists$ is used to mean "there exists".

Example

1. $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ such that $y = x^2$.
2. $\forall x \in \mathbb{R}$, $x^2 \geq 0$ or $|x| \geq 0$

Determine if the following are true or false. Justify your answer with reasoning, examples, or counterexamples, as appropriate.

1. $\exists x \in \mathbb{R}$ such that $2 < x < 3$.
2. $\nexists x \in \mathbb{R}$ such that x is both rational and irrational.
3. If x is an even number, then x can be written as $x = 2n + 1$, where n is an integer.
4. $\forall x \in \mathbb{R}$, $\forall y \in \mathbb{R}$, if $x < y$, then $2x - 1 < 2y - 1$.
5. $\forall x \in \mathbb{R}$, $\forall y \in \mathbb{R}$, if $x < y$, then $-x < -y$.

Definition (Implication)

A statement of the form *if A, then B*, which we denote

$$A \Rightarrow B.$$

Example

1. If $x < 0$, then $|x| = -x$.
   Also written as $x < 0 \Rightarrow |x| = -x$.
2. If it is raining, then there are clouds.
   Also written as rain $\Rightarrow$ clouds.

Definition (Converse)

The **converse** of a implication – if A, then B – is *if* B*, then* A, which is denoted

$$B \Rightarrow A.$$

Example

1. The converse of the statement "If x is odd, then x is not even", would be "If x is not even, then x is odd".
2. If $x \geq 2$, then $x \geq 3$. The converse of this statement would be If $x \geq 3$, then $x \geq 2$.

Definition

A **biconditional statement** is a statement of the form *A if and only if B*, which is denoted

$$A \Leftrightarrow B.$$

Examples

1. A x is odd if and only x is not divisible by 2.
2. $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}\,(xy = 0 \iff x = 0 \text{ or } y = 0)$.

Prove that the distance between two points, (x_1, y_1) and (x_2, y_2), is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Definition (Sequence)

A sequence is a list of real values that follow a specified pattern.

Consider the following sequence:

$$\left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1} \right\}$$

We can view a picture of this sequence.