LMF-L-functions and modular forms

Report on Activities June 2013-June 2015

INVESTIGATORS

Principal Investigator: Professor John Cremona, Warwick Mathematics Institute, University of Warwick, Coventry

Co-Investigators:

Dr Andrew Booker, University of Bristol
Professor Samir Siksek, University of Warwick
Professor Brian Conrey, University of Bristol
Professor Jon Keating, University of Bristol

Project Partners:
Mike Rubenstein – University of Waterloo
David Farmer – AIM
Fernando Rodriguez Villegas – ICTP
William Stein – University of Washington, Seattle

Project Researchers:
Samuele Anni –- Warwick, since October 2013
Edva Roditty-Gershon – Bristol, since August 2014
Min Lee – Bristol, since September 2014
Nicolas Mascot – Warwick, since September 2014
Aurel Page – Warwick, since September 2014
David Platt – Bristol, from September 2015

SCIENTIFIC REPORT

Since the start of the project in June 2013, significant progress has already been made on several of the research Objectives and specific Problems set out in the grant proposal, which are referred to below.

Strand L – Rigorous L-function computation package – Andy Booker

Under Strand L, three primary objectives were proposed (L1: classification of higher degree L-functions, L2: Rigorous computation of L-functions, and L3: Understanding hypergeometric motives as a source of L-functions), as well as a number of specific subproblems. Significant progress has been made on all three objectives, as detailed below, though much work remains.

To date, four LMFDB-related workshops (Bristol 2013, ICTP 2014, Oregon State 2015 and the Dublin 2015 mini-workshop) have had substantial components addressing Strand L, and another is planned for early 2016. At the Bristol workshop, specific goals for developing rigorous algorithms for computing L-functions (Objective L2) were set out, and work began on some of the research described below. During the 2015 Dublin meeting, CI Booker developed algorithms to compute the zeros of genus 2 curve L-functions and worked on the interface to those L-functions in the LMFDB.

At the Oregon State workshop, this work was broadened by PP Farmer and Koutsoliotas, who have used it as a starting point for redesigning the interface to all L-functions in the LMFDB.

In 2014, Bristol hired Dr Min Lee as a PDRA. Dr Lee is an expert on the "approximate converse theorem" approach to certifying numerical computations of higher rank automorphic forms, which is directly related to Objective L1. In ongoing joint work, Lee and CI Booker are exploring the use of the Kuznetsov trace formula as an alternative means of certifying the known GL(3) automorphic forms. This line of research is still in its early stages, but the approach seems very promising. In another direction, but also related to Objective L1, Booker and Lee are working on an extension of the Booker-Strombergsson-Venkatesh method for certifying Maass cusp forms. Besides various theoretical results, this extension will make rigorous computation of Maass forms substantially more practical than ever before.

In 2014, following initial discussions during the Bristol workshop, CI Booker and Dr Pascal Molin developed a novel method for computing high degree L-functions (Objective L2) by taking ratios with lower-degree L-functions and using explicit knowledge of their zeros. This work is still ongoing, but it has already enabled rigorous computations of some high symmetric power L-functions, for which traditional approaches are too cumbersome.

In 2015, following initial work at the 2013 workshop, CI Booker, together with Drs Jonathan Bober, Ralph Furmaniak and Min Lee, have developed new algorithms based on the Eichler-Selberg trace formula for computing holomorphic modular forms. This work will eventually have direct impact on several parts of the LMFDB. First, it will enable computing bases for spaces newforms beyond the ranges that can be reached by currently availably open-source software implementations (Sage, in particular). Second, it will enable large-scale computation of the L-functions of holomorphic modular forms (part of Objective L2). Third, it will also us to compute comprehensive data on weight 1 modular form spaces, to be included in the LMFDB.

In ongoing work, CI Booker, together with Drs Andrew Sutherland, John Voight and Dan Yasaki, have studied the ways in which the L-functions attached to genus 2 curves can arise from other sources, such as elliptic curves over quadratic fields. This work represents some of the first steps in tackling Objective L1 for degree 4 L-functions.

Bristol have recently hired Dr David Platt (starting in October 2015), who is an expert in the computational theory of L-functions. Although Platt has not officially started yet, he and CI Booker have already begun discussions on two research problems related to Strand L. The first is a rigorous algorithm for proving that all zeros of an L-function in some range of the critical line have been found; this is based on CI Booker's work on genus 2 curve L-functions and it is hoped that it will be more efficient than the traditional approach via Turing's method in many cases. The second is to carry out Objective L2 for a few specific classes of L-functions, including holomorphic modular forms and genus 2 curves.

Strand M: Bianchi newforms and elliptic curves

Research within the Modularity strand is divided into four parts, each with its own Objectives and specific Problems. Substantial progress has been made in two of these parts by the investigators at Warwick, with some progress in a third.

Modularity of elliptic curves over number fields

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Substantial progress has been made here on both theoretical and computational fronts, by PI Cremona, CI Siksek, project researcher Page and two PhD students, Koutsianas (who has partial EPSRC support) and Garcia (who has none). On Objective M2 and Problems MP1-MP3, by the start of the project existing methods for searching for elliptic curves given their conductor and traces of Frobenius were already well developed and implemented in Magma by Donnelly, and have been successfully used to compute databases of elliptic curves over five imaginary quadratic fields (using Bianchi modular forms from Cremona's database as input), many real quadratic fields (using Hilbert Modular Forms from Donelly and Voight) and one cubic field (using automorphic forms from Gunnells and Yasaki). For Bianchi forms, only 5 fields of class number 1 have so far been treated towards MP2, and Cremona and Page are developing code to extend this to other fields. In parallel, Cremona and student Koutsianas have been developing new algorithms to find (rigorously complete) sets of elliptic curves with specified primes of bad reduction, improving on earlier work by Cremona and Lingham. When finished (over the next year), this will enable us to show that the database of curves over imaginary quadratic fields is complete in the absence of any general modularity results. The work is close to completion, and Koutsianas will report on it in a workshop at ICERM in September 2015. A useful spin-off from his methods is the first ever implementation (with several new improvements) of a general S-unit equation solver over number fields, which has many other applications and has attracted considerable interest. On MP4, Garcia implemented the DGP algorithm (based on the Serre-Faltings-Livne method), and with Cremona has been developing better methods for dealing algorithmically with 2-adic Galois representations, with a view to making the algorithm more efficient. The systematic application of the algorithm to prove modularity of all the elliptic curves over imaginary quadratic fields in the database will take place when this development is complete, over the next year.

Very recent work of Freitas (Bonn), Le Hung (Harvard) and CI Siksek establishes modularity for elliptic curves over real quadratic fields. In that work it is shown that modularity for elliptic curves over a given totally real field K can be established provided that all K-rational points on some complicated modular curves can be determined and shown to be modular. These are generally curves for which the Mazur and Merel formal immersion criterion fails. Mascot and Siksek have started an ambitious project to determine points on modular curves using a fundamental domain as a replacement of an algebraic model of the curve, and using rigorous computations of integrals as an alternative to the formal immersion criterion. Once complete this project should yield a proof of modularity for elliptic curves over many totally real fields.

Bianchi modular forms of higher weight and mod-p Bianchi forms

Objectives M1 and M3 are in progress by Cremona and Page (who was appointed during year 2). Page is also redeveloping the database of automorphic forms over number fields to handle general field signatures (currently only totally real fields are supported). Objective M5 relies on input by Sengun, who moved from Warwick to Sheffield in 2014 and is remains an active collaborator.

Modularity of Abelian surfaces and the paramodular conjecture

Warwick EPSRC fellow Dembélé has made progress on Objective M6 and Problem MP6. With Abhinav Kumar (MIT), he has provided examples of paramodular abelian surfaces B defined over Q, which become of GL_2-type over real quadratic fields [LD1]. With Tobias Berger (Sheffield), Ariel Pacetti (Buenos Aires) and Haluk Sengun (Warwick/Sheffield), he also showed how a similar strategy

can be adapted to imaginary quadratic fields; in this case, proving modularity is much harder, but can be done using the Faltings-Serre method [LD2].

[LD1] L. Dembélé, A. Kumar: Examples of abelian surfaces with everywhere good reduction, Math. Annalen (online, July 2015), DOI 10.1007/s00208-015-1252-6.

[LD2] T. Berger, L. Dembélé, A. Pacetti, M. H. Şengün: Theta lifts of Bianchi modular forms and applications to paramodularity, J. London Math. Soc. (online, July 2015), DOI 10.1112/jlms/jdv023.

Applications of modular form computations

(a) to Diophantine equations

Freitas (Bonn) and CI Siksek have carried out a detailed study of the Fermat equation over totally real fields, investigating the generalisations of the ideas of Frey, Serre, Ribet and Wiles. This work has lead to two substantial papers. The first establishes an asymptotic version of Fermat's Last Theorem for five-sixth of real quadratic fields. The second paper builds on the theory developed in the first paper and on heavy computations of modular forms over real quadratic fields to establish the complete FLT over many real quadratic fields of small discriminant.

Project researcher Anni and CI Siksek have investigated generalized Fermat equation of signature (2n,2m,p) and shown how to associate this to a Frey curve over the maximal real subfield of the p-th cyclotomic field. Using level lowering arguments and modular form computations over these fields, they solve this equation for p between 3 and 13. Their paper also contains a new modularity theorem for semistable elliptic curves over real abelian fields.

The work of Freitas, Anni and Siksek on problem MP7 will be used as blueprint for tackling similar problems over imaginary quadratic fields. This is currently being carried out by Sengun (Sheffield) and Siksek.

(b) to Galois Theory

Work on problem MP8 is being carried out by researcher Anni and CI Siksek. For now the problem is being investigated in the easier setting of elliptic curves (over number fields) and abelian varieties. Anni and CI have established a uniformity theorem for semistable elliptic curves over totally real fields that is analogous to Serre's theorem for semistable elliptic curves over the rationals. In joint work by Anni, Lemos (Warwick PhD student) and Siksek, it is shown that the mod l representation of a principally polarized abelian variety, satisfying some further mild conditions and with l sufficiently large, is either reducible or surjective. This is used to show that the degree 6 mod l general symplectic group can be explicitly realized as a Galois group over the rationals.

Strand S – Moment conjectures for degree 4 L-functions

One of the central problems in the theory of *L*-functions is the calculation of moments. Using traditional number-theoretic techniques, based on averaging Dirichlet polynomials, it is usually possible to prove asymptotic formulae for the first few moments and to conjecture a few more; for example, in the case of the Riemann zeta function the first moment was computed by Hardy & Littlewood in 1918, the second by Ingham in 1926, the third was conjectured by Conrey and Ghosh in

1991, and the fourth by Conrey & Gonek in 2000. It is striking that traditional techniques fail after the first few moments; and this is not just a matter of rigour, even at the level of conjectures they give nonsensical answers (e.g. negative when the moments are non-negative by definition). One of the notable successes of Random Matrix Theory has been a detailed and precise set of conjectures for all of the moments. But this leaves open the question as to why traditional methods fail. We (Conrey & Keating) now believe we understand how to refine these methods so they give answers that agree with the random matrix conjectures for all moments. This is a major research programme that will take some time to complete, because the calculations are quite intricate and new computational methods are required. Three papers initiating this programme have been written [18,19, 20]. We believe that once this programme is completed it will represent a substantial step forwards.

In recent years, following on from the seminal work of Katz & Sarnak, there has been a significant focus on connections between function-field *L*-functions and Random Matrix Theory. Keating & Rudnick [27] used this approach to calculate the mean and variance of sums of the Möbius function and its square, in both short intervals and arithmetic progressions, in the context of the ring of polynomials over a finite field of q elements in the limit as q tends to infinity. This is done by relating the variances to matrix integrals, using recent equidistribution results due to Katz. In many cases the results mirror what is known or conjectured in the number field setting, but in other cases there are subtle and surprising differences. This approach was extended to divisor functions in [28]; previously almost nothing was known, and our results suggest a complicated and unexpected structure in this case. It was also applied in [26] to determine the mean-square error in the function-field analogue of the Hardy-Littlewood conjecture, proved in this setting by Bary-Soroker, and its generalizations. The results prove that recent speculations about the size of this error cannot be correct.

Montgomery's original conjecture concerning the pair correlation of the zeros of the Riemann zeta-function and the Gaussian Unitary Ensemble of Random Matrix Theory was shown by Goldston & Montgomery to be equivalent to a conjecture concerning the variance of the number of primes in short intervals. This was extended to all *L*-functions in the Selberg class in [17]. One of the main findings was that the variances of the prime sums that arise in this context have a different form when the degree of the associated *L*-functions is 2 or higher to that which holds when the degree is 1 (e.g. the Riemann zeta-function). Specifically, when the degree is 2 or higher there are two regimes in which the variances take qualitatively different forms, whilst in the degree-1 case there is a single regime.

The problem of determining the probability that a random integral quadratic form in n variables has an integral zero was addressed in [8]. When real quadratic forms in n variables are distributed according to the Gaussian Orthogonal Ensemble (GOE) of Random Matrix Theory, the probability that a random such real quadratic form is isotropic (i.e., indefinite) was determined explicitly by combining methods from number theory with the evaluation of certain matrix integrals. This interplay between the different strands of the Programme Grant was unanticipated, and, we believe, is a good example of what such grants make possible.

PUBLICATIONS

1) Samuele Anni, A local-global principle for isogenies of prime degree over number fields, Journal of the London Mathematical Society, Issue 3, Volume 89
2) Samuele Anni and Samir Siksek, On Serre's uniformity conjecture for semistable elliptic curves over totally real fields, Mathematische Zeitsschrift, 2015.
3) Samuele Anni and Samir Siksek, On the generalized Fermat equation $x^{2l}+y^{2m}=z^{p}$ for $3\leq p\leq 13$.

4) Samuele Anni. Pedro Lemos, Samir Siksek, Residual Representations of Semistable Principally Polarized Abelian Varieties. Preprint.
5) Barinder Banwait, John Cremona, Tetrahedral Elliptic Curves and the local –to-global principle for isogenies, Algebra & Number Theory, Volume 8, 2014, pp 1201-1229.
6) Michael Bennet, Sander Dahman, Maurice Mignotte and Samir Siksek, Shifted powers in binary recurrence sequences. Mathematical proceedings of the Cambridge Philosophical Society, Volume 158, 305-329.
7) Manjul Bhargava, John Cremona, Tom Fisher, The proportion of plan cubic curves over Q that everywhere locally have a point, International Journal of Number Theory.
8) Manjul Bhargava, John Cremona, Tom Fisher, Jon Keating, Nick Jones, What is the probability that a random integral quadratic form in n variables has an integral zero? IMRN
9) A.R. Booker and F. Thorne, "Zeros of L-functions outside the critical strip", Algebra Number Theory 8 (2014), no. 9, 2027–2042.
10) A.R. Booker, G.A. Hiary and J.P. Keating, "Detecting squarefree numbers", Duke Math. J. 164 (2015), no. 2, 235–275.
11) A.R. Booker, "L-functions as distributions", Math. Ann. 363 (2015), no. 1-2, 423–454.
12) A.R. Booker, "Simple zeros of degree 2 L-functions", to appear in JEMS.
13) A.R. Booker and M. Krishnamurthy, "A converse theorem for GL(n)", submitted.
14) A.R. Booker and S. A. Irvine, "The Euclid-Mullin graph", submitted.
15) A.R. Booker and M. Lee, "The Selberg trace formula as a Dirichlet series", submitted.
16) Andrew Bremner, Samir Siksek, Squares in arithmetic progression over cubic fields, International Journal of Number Theory.
17) H.M.Bui, J.P. Keating & D.J.Smith, on the variance of sums of arithmetic functions over primes in short intervals and pair correlation for L-functions in the Selberg class, submitted for publication.
18) B.Conrey & J.P. Keating, Moments of zeta and correlations of divisor-sums: 1, Phil.Trans. R. Soc, 2015.
19) B. Conrey & J.P. Keating, Moments of zeta and correlations of divisor-sums: II, accepted for publication in Advances in the Theory of Numbers, Fields Institute Communications 77 (Springer)
20) B. Conrey & J.P. Keating, Moments of zeta and correlations of divisor-sums: III, accepted for publication in Indagationes Mathematicae,
21) John Cremona and Maite Aranes, Congruence subgroups, cusps and Manin symbols over number fields, Contributions in Mathematical and Computational Sciences, 2014.
22) Nuno Freitas, Bao Le Hung, Samir Siksek, Elliptic Curves of Real Quadratic Fields are Modular, Inventiones Mathematicae, Volume 201, 2015, pp 159-206.
23) Nuno Freitas, Samir Siksek, An Asympiotic Fermat's Last Theorem for Five Sixths of Real Quadratic Fields, Compositio Mathematica, Volume 151, 2015, pp 1395-1415
24) Nuno Freitas, Samir Siksek, Fermat's Last Theorem over some small real quadratic fields, Algebra & Number Theory, Volume 9, 2015, pp 875-895.
25) J.P. Keating, Z. Rudnick & T.W. Wooley (2015), Number fields and function fields: coalescences, contrasts and emerging applications**,** *Phil. Trans. R. Soc. A 373, 20140315* (DOI: 10.1098/rsta.2014.0315)
26) J.P. Keating & E. Roditty-Gershon, Arithmetic correlations over large finite fields, accepted for publication in IMRN (it has already appeared online, but not yet in print - see http://imrn.oxfordjournals.org/content/early/2015/06/04/imrn.rnv157.full.pdf+html?sid=212576a5-10dd-4cf9-baa6-e0daab6e263f)

27) J.P. Keating & Z. Rudnick, Squarefree polynomials and Möbius values in short intervals and arithmetic progressions**,** submitted for publication.
28) J.P. Keating, B. Rodgers, E. Roditty-Gershon & Z. Rudnick, Sums of divisor functions in Fq[t] and matrix integrals, submitted for publication.
29) Samir Siksek, Every integer greater than 454 is the sum of at most seven positive cubes, preprint, http://arxiv.org/abs/1505.00647.

RECRUITMENT AND CAREER DEVELOPMENT OF RESEARCH STAFF

The project proposal planned for six 3-year post-doctoral researchers to be appointed, three each in Warwick and Bristol, to work with the PI and CIs on different aspects of the project. All these appointments have been made, and (as of August 2015) all but one has taken up their position.

Recruitment

All posts were advertised widely and internationally, and attracted an excellent number of well qualified applicants from across Europe and North and South America. After short-listing, interviews were carried out by teleconferencing. The Warwick and Bristol teams consulted each other during this process, so that applicants who had applied for positions in both places could be appointed to the place most appropriate to their expertise and preference. This recruitment process started early in Year 1 and was complete by the middle of Year 2. One researcher started in Warwick during Year 1 and two more in Year 2; two started in Bristol in Year 2 and the third will start in Year 3. The six individuals appointed range in experience from recent PhDs to having had one or two previous post-doctoral positions, and between them cover all three research strands.

Career development

All the researchers appointed have already made substantial contributions to the research of the project, as can be seen from the detailed research reports in this document.

In the first six months after taking up their appointments, research staff at both Bristol and Warwick have a probationary status and have several appraisal meetings with their manager (PI Cremona and Co-Is Booker and Keating); at the end of this period their appointments have all been enthusiastically confirmed. Further appraisal meetings will take place at least annually. As each researcher starts their third year of appointment, guidance and advice on the next step of their academic careers will be given: so far none of the researchers has reached this stage. Each researcher is encouraged, and given advice (where necessary) on publication of their research results. In Warwick, since October 2014 when the full complement of researchers was in post, PI Cremona has held weekly group meetings with them at which specific plans for the work to be carried out are made, and research is carried out. These meetings have been particularly useful regarding the development of the LMFDB database. Similarly, in Bristol, Booker and Keating have met regularly with the researchers based there.

All the appointed researchers have made use of the travel funds available to them from the grant to attend relevant conferences and make research visits to collaborators. From September to December 2015, the researchers in Warwick will accompany PI Cremona to attend the semester-long programme “Computational Aspects of the Langlands Programme" at ICERM (Brown University, Providence, RI, USA), at which a large number of contributors to the wider LMFDB project will be present, which is

expected both to result in substantial advancements in several of the project's research areas and also, through the many personal contacts which will be made there, to advance the researchers' own career prospects. The Bristol team will also participate in the ICERM Programme.

PARTIALLY OR WHOLLY FUNDED VISITING RESEARCH COLLABORATORS

The following collaborators have visited either Bristol or Warwick since June 2013 to collaborate on aspects of the research programme. Some of these have been funded in whole or in part by other sources.

Nuno Freitas	Bayreuth	One week – November 2013	Warwick
Marc Masdeu	Columbia	One week – November 2013	Warwick
Nuno Freitas	Bayreuth	One week - July 2014	Warwick
Manjul Bhargava	Princeton	10-20 October 2014	Warwick
Tom Fisher	Cambridge	11-12 & 17-19 October 2014	Warwick
Ariel Pacetti	Buenos Aires	7-16 May 2015	Warwick
Chantal David	Montreal		Bristol
Chris Hall	Wyoming		Bristol
Nick Katz	Princeton		Bristol
Elizabeth Meckes	Case Western		Bristol
Mike Rubinstein	Waterloo		Bristol
Leonid Pastur	Kharkov		Bristol
Paul Pollack	Georgia		Bristol
Brad Rogers	Zurich		Bristol
Zeev Rudnick	Tel Aviv		Bristol
German Sierra	Madrid		Bristol
Jaehyun Cho	Buffalo	13-19 April 2014	Bristol
Myoungil Kim	Connecticut	13-19 April 2014	Bristol
Micah Milinovich	Mississippi	14-22 May 2014	Bristol
Michael Jacobson	Calgary	25 May-1 June 2014	Bristol
Pascal Molin	Paris	7-28 June 2014	Bristol
Muthu Krishnamurthy	Iowa	26 June-11 July 2014	Bristol
Min Lee	Brown	28 June-10 July 2014	Bristol
Masao Tsuzuki	Sophia	6-12 July 2014	Bristol
Shunsuke Yamana	Kyushu	6-12 July 2014	Bristol
Nathan Ng	Lethbridge	7-12 December 2014	Bristol

UNFUNDED RESEARCH COLLABORATORS

The research carried out by Dr Lassina Dembélé (Warwick) for his EPSRC CAF includes work relevant to Strand M and has been reported on above. Dr Marc Masdeu was appointed to work at Warwick with Dembélé in 2014-2015, and from 2015-2017 he has a Marie-Curie Fellowship at Warwick.; he will also be contributing to the LMF Project.

EQUIPMENT

Purchase, installation and configuration of computing equipment
The Warwick equipment (3 servers) was purchased and installed in 2013; since September 2013 it has hosted the main LMFDB website and database, and is now being extensively used. Bristol also have their equipment (5 servers, part funded by other grants) installed and functioning.

RESEARCH WORKSHOPS AND OTHER EVENTS

Bristol LMFDB Workshop, 9-22 September 2013

Workshop themes:

1. (September 9-15) Core development of LMFDB pages. Particular goals are completing all remaining work for L-functions of degree at most 2 and designing pages for Hecke character L-functions.
2. (September 16-22) Development of algorithms for arbitrary-precision and rigorous evaluation of high-degree L-functions. Our ultimate goal is to allow the user to evaluate any numerical parameters appearing on LMFDB pages, including special values, zeros, etc., to guaranteed accuracy and high precision where feasible.

Related NSF workshop: *Curves and Automorphic Forms*, Arizona State University, 10-14 March 2014. Co-organised by David Farmer (Project Partner, AIM) with John Jones (Arizona Stata), Paul Gunnells (University of Massachusetts, Amherst) and Holly Swisher (Oregon State).

Warwick LMFDB Workshop, 2-6 June 2014

Workshop themes: Bianchi and Hilbert Modular Forms, elliptic curves over number fields, LMFDB development.

ICTP LMFDB Workshop: *Workshop on L-functions and Modular Forms*, ICTP Trieste, 7-12 September 2014. Co-organised by PI Cremona, Fernando Rodriguez Villegas (Project Partner, Abdus Salam ICTP) and M. Vlasenko (UC, Dublin).

Related NSF workshop: *Computational Representation Theory in Number Theory*, July 27-31, 2015 Oregon State University Corvallis, Oregon, USA. Co-organised by PI Cremona with David Farmer (Project Partner, AIM), John Jones (Arizona Stata), Paul Gunnells (University of Massachusetts, Amherst) and Holly Swisher (Oregon State).

PLANNED RESEARCH ACTIVITIES

LMF team members were involved in the organisation of and participated in two workshops, independently funded by the US National Science Foundation (NSF) in 2014 and 2015, both of which were on topics closely related to the project and included hands-on development work on our database and website.

Similarly, in September-December 2015 a semester-long programme on "Computational aspects of the Langlands Programme" will take place at ICERM (the Institute for Computational and Experimental Mathematics, Brown University, Providence RI, USA). The organisers of the programme include CI Brian Conrey and Project Partners David Farmer (AIM) and Mike Rubinstein (Waterloo), and all but one of the other organisers were invited participants at the June 2014 Warwick workshop. Many of the topics of this programme are closely related to LMF research goals, including those of the three workshops during the semester, and all the researchers working on the LMF project will take part in the semester's activities: PI John Cremona will be there for the entire semester and is a co-organiser of the opening workshop on "Computational aspects of modular forms and curves of small genus over number fields" and Partner Mike Rubinstein is a co-organiser of one of the later workshops. It was therefore decided to fit our own workshops around these activities, and not hold a workshop of our own during that semester, but instead to make use of our resources to enable the LMF researchers to attend the semester's workshops and other activities as appropriate.

The AIM workshop originally envisaged for spring 2014 will not now take place until 2016. There will also be a workshop in Bristol in March 2016, to fit around the 2016 BMC hosted there, and we plan to have a meeting with members of the Scientific Steering Committee during that time (one of its members, Professor Sarnak from Princeton, will be a plenary speaker at the BMC).

The most appropriate use of the 2016 workshops is still under discussion. This could be on the technical side of LMFDB, to look at systems and functionality of the database, in order to review developments with the database and ensure long term viability. Recent developments in Degree 4 L-functions will also be a suitable topic to include in that workshop.

A further workshop is planned for 6-10 June 2016, to held jointly with the London Langlands Programme grant, coorganised by Fred Diamond and Wansu Kim at Kings College.

TALKS

From the start of the grant to June 2015, group members gave the following research talks, minicourses, or full research courses:

Date	Researcher	Title/occasion	Venue
June 2013	Andy Booker	"Alan Turing and the Riemann hypothesis"	RIMS
June 2013	Andy Booker	"Converse theorems, explicit formulae and the Selberg class"	RIMS
June 2013	Andy Booker	"Simple zeros of degree 2 L-functions"	Nagoya
June 2013	Jonathan Keating	Invited lecture, Branching Diffusions and Gaussian Free Fields in Physics, Probability and Number Theory	Marseille, June 2013
July 2013	Andy Booker	"Alan Turing and the Riemann hypothesis"	Sophia
July 2013	Samir Siksek	“Modularity over Real Quadratic Fields”, Explicit Methods in Number Theory	Oberwolfach

LMF-L-functions and modular forms

Report on Activities June 2013-June 2015

July 2013	Samuele Anni	“Image of residual modular Galois representations: an algorithm”, Sage Days: Algorithms in Arithmetic Geometry.	Lorentz Center, Leiden
July 2013	Samuele Anni	“ A local-global principle for isogenies between elliptic curves over number fields”, 28th Journées Arithmétiques,	Grenoble
July 2013	Samir Siksek	“The Fermat Equation over Totally Real Fields”, Rational Points 2013	Schloss Thurnau
September 2013	Jonathan Keating	Invited Keynote Lecture, Workshop on Number Theory and Physics, Clay Mathematics Institute.	University of Oxford
September 2013	Samir Siksek	“The Fermat Equation over Totally Real Fields”	Debrecen, Hungary.
September 2013	Samir Siksek	“Rational Points on Curves” (minicourse), Algebraic and Explicit Methods in Number Theory	Sallins-les-Bains, Jura, France
October 2013	Andy Booker	"WTF is an L-function?"	Linfoot
October 2013	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Number Theory Seminar	University of Sheffield
October 2013	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Number Theory Seminar	University of Cambridge
November 2013	Jonathan Keating	Distinguished Lecture Series, Baylor University.	Texas USA
November 2013	Jonathan Keating	Workshop on Non-equilibrium Dynamics and Random Matrices, Institute for Advanced Study, Princeton, USA, November 2013 (Invited Lecture).	
December 2013	Jonathan Keating	Invited lecture, Conference on Random Matrix Theory.	Bielefeld, Germany
December 2013	Samuele Anni	“ Residual modular Galois representations and their images”, Number Theory Seminar	University of Warwick
December 2013	Samuele Anni	“ Minimal Models and Integral Models”, Algebraic Geometry for Number Theory (TCC)	University of Warwick
December 2013	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Séminaire d'Arithmétique et de Géométrie Algébrique	Université Paris-Sud, Orsay
January 2014	Jonathan Keating	Invited Keynote Lecture, Workshop on Arithmetic Statistics over Finite Fields and Function Fields, American Institute of Mathematics	
January 2014	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Number Theory Seminar	University of Oxford
March 2014	John Cremona	"Tables of elliptic curves over number fields", Curves and Automorphic Forms workshop	Arizona State University
March 2014	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Curves and Automorphic	Tempe, Arizona

		Forms	
May 2014	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Séminaire de théorie des nombres	de l'IMJ-PRG, Paris
July 2014	Samuele Anni	“Twists and residual modular Galois representations, Building Bridges	University of Bristol
July 2014	Samir Siksek	“The Fermat Equation over Totally Real Fields”, Building Bridges: 2nd EU/US Workshop on Automorphic Forms and Related Topics	University of Bristol
July/August 2014	Samir Siksek	“Modularity and the Fermat Equation over Totally Real Fields”, XIII Escola de Algebra	Maringa, Brazil
August/September 2014	Samir Siksek	“Rational Points on Curves” (minicourse), Arithmetic of Hyperelliptic Curves and Cryptography	Ohrid, Macedonia
September 2014	Jonathan Keating	Invited Lecture, Workshop on Statistics and Number Theory, Centre de Recherches Mathématiques.	Montreal, Canada,
September 2014	Samuele Anni	“ Residual modular Galois representations: an algorithmic approach”, Algebra and Number Theory Seminar	University College Dublin
September 2014	Aurel Page	"Computing Klein modular forms", WANDS seminar.	University of Sheffield
September 2014	Samir Siksek	“Modular Techniques for Diophantine Equations” (minicourse), Introductory Workshop on Modular Methods in Diophantine Equations	Bilecik, Turkey
October 2014	Samuele Anni	“ On Serre's uniformity conjecture for elliptic curves”, Heilbronn Seminar	University of Bristol
October 2014	Min Lee	“Shifted multiple Dirichlet series and spectral moments of Rankin- Selberg L-functions”, Number theory and geometric seminar	University of Nottingham
October 2014	Min Lee	“Second Moments and simultaneous non-vanishing of GL(2) automorphic L-series”, Number Theory seminar	Ohio State University,Columbus, OH, USA
Octoebr 2014	Aurel Page	Computing Klein modular forms", number theory seminar.	University of Warwick
November 2014	Samuele Anni	“ Residual Galois representations: a database”, Sheffield Number Theory Seminar	University of Sheffield
November 2014	Min Lee	“Second Moments and simultaneous non-vanishing of GL(2) automorphic L-series”, Linfoot seminar	University of Bristol
November	Aurel Page	"Computing Klein modular forms",	University of

2014		Heilbronn seminar.	Bristol
December 2014	Samuele Anni	“Twists and local data of residual modular Galois representations”, Number Theory Seminar	Universidad de Buenos Aires
December 2014	John Cremona	"The L-functions and modular forms database project", Foundations of Computational Mathematics (invited plenary speaker)	Universidad de la República, Montevideo, Uruguay
December 2014	John Cremona	"Some density results in number theory", School in Computational Algebra and Number Theory.	Universidad de la República, Montevideo, Uruguay.
December 2014	Jonathan Keating	Invited Lecture, Analysis, Spectra and Number Theory – Conference Honoring Peter Sarnak	Princeton, USA
December 2014	Min Lee	“Second Moments and simultaneous non-vanishing of GL(2) automorphic L-series”, NCTS, NCTS-PMI joint workshop in Number Theory	Hsinchu, Taiwan
December 2014	Min Lee	“Shifted multiple Dirichlet series and spectral moments of Rankin- Selberg L-functions”, BIRS, Families of Automorphic Forms and the Trace Formula (14w5120),	Banff, Canada
December 2014	Aurel Page	"Calcul de formes modulaires de Klein", number theory seminar.	Université de Clermont-Ferrand, France
January 2015	Andy Booker	Detecting squarefree numbers"	A & M, Texas
January 2015	John Cremona	"Some density results in number theory", Number Theory Seminar	University of Warwick
January 2015	Aurel Page	"Tutorial: associative and central simple algebras", PARI/GP Workshop	Université de Bordeaux, France.
January 2015	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Number Theory Seminar.	University of Barcelona
February 2015	Andy Booker	"L-functions as distributions"	Oxford
February 2015	Andy Booker	"L-functions as distributions"	Gottingen
February 2015	Andy Booker	"L-functions as distributions "	London
February 2015	Andy Booker	"L-functions as distributions"	University of Cambridge
February 2015	Jonathan Keating	Invited Lecture Series, Courant Institute.	New York,
February 2015	Samir Siksek	“Elliptic Curves over Real Quadratic Fields are Modular”, Number Theory Seminar	MPIM, Bonn
February 2015	Edva Roditty-	“Arithmetic Statistics in Function Fields” Linfoot Seminar	University of Bristol

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March 2015	Samuele Anni	"Some density results in number theory", Oxford Number Theory Seminar.	University of Oxford
March 2015	John Cremona	"Some density results in number theory", Number Theory Seminar	University of Oxford
March 2015	John Cremona	"Some density results in number theory", invited speaker at `Elliptic Curves, Modular Forms and Iwasawa Theory' conference in honour of the 70th birthday of John Coates	University of Cambridge
March 2015	Aurel Page	"Aspects algorithmiques des groupes d'unités", ATI seminar.	Institut de Mathématiques de Marseille, France
March 2015	Edva Roditty-Gerson	"Arithmetic Statistics in Function Fields" Number Theory Seminar	University of Tel Aviv, Israel
March 2015	Samir Siksek	"Elliptic Curves over Real Quadratic Fields are Modular", Number Theory Seminar	University of Exeter
March 2015	Samir Siksek	"Modularity and the Fermat Equation over Totally Real Fields", Algebra Seminar	University of Birmingham
April 2015	Andy Booker	"Alan Turing and the Riemann Hypothesis"	Kobe
April 2015	Andy Booker	"On decidability of Artin's conjecture"	Kobe
May 2015	Samuele Anni	" Residual modular Galois representations: images and applications" Occasion/venue: London Number Theory Seminar,	King's College London
May 2015	Samuele Anni	"On Serre's uniformity conjecture for elliptic curves" Occasion/venue: Number Theory Seminar	University of York
May 2015	Min Lee	"The Selberg trace formula as a Dirichlet series", PMI Number theory seminar	POSTECH , Pohang, Republic of Korea
June 2015	Jonathan Keating	Invited Plenary Lecture, International Conference on Number Theory and Physics.	Rio de Janeiro.
June/July 2015	Samir Siksek	"On the Diophantine Equation x^2l+y^2m=z^p", Rational Points 2015 ,	Franken-Akademie, Schloss Schney
July 2015	John Cremona	"Some density results in number theory", Explicit Methods in Number Theory	Oberwolfach, Germany

LMF-L-functions and modular forms

Report on Activities June 2013-June 2015