$\Gamma_0(N)\begin{pmatrix}a&b\\c&d\end{pmatrix}$ $\Updownarrow$ $(c:d)$

{right cosets of $\Gamma_0(N)$ in $SL_2(\mathbb{Z})$} $\Updownarrow$ Prop

$\{x_1,\dots,x_n\} = \mathbb{P}^1(\mathbb{Z}/N\mathbb{Z}) \longrightarrow H_1(X_0(N),\mathbb{Q},\{\text{cusps}\})$

$\begin{pmatrix}*&*\\0&*\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}$

$= \{(u,v) : 0\le u,v\le N-1,\ \gcd(u,v,N)=1\} / \underset{(\mathbb{Z}/N\mathbb{Z})^\times}{\sim}$

(e.g. $\mathbb{P}^1(\mathbb{Z}/p\mathbb{Z}) = \{(0:1),(1:0),(1:1),(1:2),\dots,(1:p-1)\}$

$\langle \underbrace{S=\begin{pmatrix}0&-1\\1&0\end{pmatrix}}_{\text{order }4},\ \underbrace{T=\begin{pmatrix}1&-1\\1&0\end{pmatrix}}_{\text{order }6} \rangle = SL_2(\mathbb{Z})$

$(u,v) \qquad (u+v,-u)$

$x_i=(u:v)$

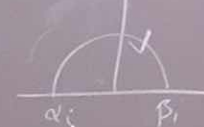
$x_i + x_iS = 0 \longleftrightarrow (u:v)+(v:-u)=0$

$x_i + x_iT + x_iT^2 = 0 \longleftrightarrow (u:v)+(u+v:-u)+(v:-u-v)=0$

new born

$\alpha_i,\beta_i\in\mathbb{P}^1(\mathbb{Q})$

$\sum a_i\{\alpha_i,\beta_i\}$



$\begin{pmatrix}a&b\\c&d\end{pmatrix}\{0,\infty\} = \left\{\frac{b}{d},\frac{a}{c}\right\}$

$\gamma(x_i)-x_i=0$

Why these relations?

$2\dim S_2 + \dim Eis_2 = \dim S_2 + \dim M_2$

$(u,v)\longmapsto\{\alpha,\beta\}$

$T_p(\{\alpha,\beta\})$

$= \sum_{0\le a\le p-1}\begin{pmatrix}1&a\\0&p\end{pmatrix}\{\alpha,\beta\}$

$+ \begin{pmatrix}p&0\\0&1\end{pmatrix}\{\alpha,\beta\}$

$H_1(X_0(N),\mathbb{Q},\{\text{cusps}\})$

$\cong \mathbb{Q}[\mathbb{P}^1(\mathbb{Z}/N\mathbb{Z})] / S,T$ relations.

		dim M	dim S			
Travis & Manor	– $102 = 2\cdot 3\cdot 17$	22	15	1+1+1	8	37
Gerardo & Kevin	– $144 = 2^4\cdot 3^2$	36	13	1+1	24	49
William & Yannick	– $150 = 2\cdot 3\cdot 5^2$	42	19	1+1+1	24	61

$(\mathbb{F}_2)^n = V$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \end{bmatrix}$$

, sign=1)

```
a = M.hecke_matrix(7)
print a.str()        show(a.fcp())
```

Joe Wetherell

$S_2(\Gamma_0(p))$ $p = 389$

$p \sim 250$

$p \mid \mathrm{disc}(\mathrm{charpoly}(T_\ell))$

$\ell = 2, 3, 5, \ldots$