

$$S_2(\Gamma_0(43)) \subseteq M_2(\Gamma_0(43))$$

3-diml.

codim 1

$$M_2(\Gamma_0(1)) = 0$$

$\dim(M_k(\Gamma_0(1))) = 1$ for $k \geq 4$ even.

"modular curves"

$$\Gamma_0(Np) < \Gamma_0(N)$$

(Katz-Mazur / Z)

newform

$$X_0(Np)$$

T_p

$$X_0(N)_{/\mathbb{Q}}$$

$$X_0(N) = \Gamma_0(N) \backslash \mathbb{H}^* = \text{---}$$

"correspondence"

induces every Hecke operator...

$$\begin{aligned} \text{genus}(X_0(N)) &= \dim H^0(X_0(N), \Omega) \\ &= \dim S_2(\Gamma_0(N)) \sim \frac{N}{12} \end{aligned}$$

f_1, f_2, f_3

$$S_2(\Gamma_0(43))_{\text{new}}$$

$$f = \sum_{n=1}^{\infty} a_n q^n$$

$a_1 = 1$

$$\Pi = \mathbb{Z}[T_1, T_2, \dots]$$

I

newform = eigenform for all Π in the new subspace with coeff of $q^1 = 1$

$$f_1 = q - 2q^2 + \dots$$

$$f_2 = q + \sqrt{2}q^2 + \dots$$

$$f_3 = q - \sqrt{2}q^2 + \dots$$

$$\frac{S_2(\Gamma_0(N)) \times H_1(X_0(N), \mathbb{Z})}{(\mathcal{F}, \gamma)} \rightarrow \mathbb{C}$$

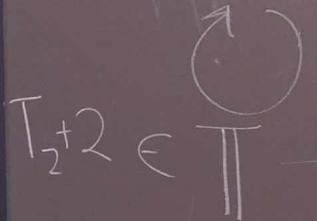
$\int_{\gamma} f_2 dz$
 $V_{\mathbb{C}}$ - vector space of dim g
 $\Lambda =$ free ab group of rank $2g$.

Abel

1800s

$$J_0(43) \stackrel{\text{defn}}{=} \text{Jac}(X_0(43))$$

$$J_0(43)(\mathbb{C}) = \frac{V}{\Lambda} = \frac{\text{Hom}(S_2, \mathbb{C})}{\text{Hom}(H_1(X_0(43), \mathbb{Z}), \mathbb{C})}$$



- abelian variety / \mathbb{C}

$$T_2 + 2 \in \mathbb{T} \rightarrow \text{End}(J_0(43)) \rightarrow \text{dimension} = \text{genus}(X_0(43)) = 3$$

$$J_0(43)(\mathbb{C}) \cong \text{Div}^0(X_0(43)) = \left\{ \sum_{p \in X_0(43)(\mathbb{C})} n_p P \mid \sum n_p = 0 \right\}$$

finite sum

$$T_2 f_1 = -2f_1$$

$$T_2 f_2 = \sqrt{2}f_2$$

ker $(T_2 - \sqrt{2})$

$$A \sim A_1 \times A_2 \times \dots \times A_n$$

simple

$$\mathbb{D} \rightarrow J_0(43) \rightarrow E \times A_{f_2} \rightarrow 0$$

43a dimension 2
 $\mathbb{Z}[\sqrt{2}] = \text{End}(A_f)$

finite group

Theorem (Shimura) $f \mapsto A_f / \mathbb{Q}^*$

$$L(A_f, s) = \prod_{\sigma: K_f \rightarrow \mathbb{C}} \sum \frac{a_n^\sigma}{n^s}$$

Sketch Proof: $f \in S_2(\Gamma_0(N))$

$\sum a_n q^n$ newform \mathcal{O}_T

$$I_f = \text{Ann}(f) \subseteq \mathbb{T} \rightarrow \text{End}(J_0(N))$$

$$= \{t \in \mathbb{T} \mid t \cdot f = 0\}$$

$$\mathbb{T}/I_f \cong \mathbb{Z}[a_n(f)]$$

eigenvalue

an order in a number field K_f

$$A_f = \left[\begin{array}{c} J_0(N) \\ I_f J_0(N) \end{array} \right]$$

$\dim(A_f) = [K_f : \mathbb{Q}]$