Final Exam: Computer-algebra portion

COMP 150

Spring 2021

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(0a) Please type your name at the top of the worksheet as a comment.

(0b) The policy for the exam is as follows: Do your exam independently. Do not engage in consultation with others. (Exception: You are encouraged to ask me questions in the public chat, the private chat, or in a breakout.) During the exam, you may use your class notes, code you wrote previously, Sage and CoCalc, and resources posted on our Blackboard site. You are not permitted to engage with any other resources of any kind anywhere. Please confirm your compliance with the academic honesty policy for this quiz and your understanding of the consequences for violations. You can do this by typing I confirm. I understand..

(1) Define the function $f(x) = e^{-x^2}$, sometimes written equivalently as $f(x) = \exp(x^2)$, for use throughout this worksheet. Calculate the second derivative of $f(x)$ (the derivative of the derivative) using Sage. Call it fpp(x). (Remember we can differentiate the function $h(x)$ in Sage using the syntax derivative(h(x), x) or using the calculus menu.)

(2) Graph fpp(x) on the interval $0 \leq x \leq 2$. (Recall that fpp(x) gives the concavity of the graph of $f(x)$.)

(3) What is the largest value (an upper bound) M that the absolute value of fpp(x) could be on the interval $0 \leq x \leq 2$ (meaning what is the farthest distance from the x-axis that the graph of the second derivative of $f(x)$ can get)? Explain your answer by referring to the graph.

(4) When Simpson's Rule is used to estimate the area under a curve given by $y = f(x)$ on an interval $[a, b]$, the error (difference) between the numerical estimate and the exact area A is known to be at most $E = \frac{(b-a)^3}{12n^2}M$, where M is any upper bound on $|f''(x)|$ on $[a, b]$, and n is a number of subintervals used in the estimation. For $f(x) = e^{-x^2}$ on the interval $[0, 2]$, where $a = 0$ and $b = 2$, find an exact value of E if $n = 6$. (Use your work in (3) to help you.) Find a decimal approximation for E, as well.

(5) Consider a numerical estimate for the area under a curve given by $y = h(x)$ on an interval $[a, b]$. Suppose the numerical estimate is 0.4788 and the exact area is A. If we know the error (difference) between the estimate 0.4788 for the area and the exact area A is at most 0.1, could the

exact area be $A = 0.55$? Why or why not? What is the biggest the exact area A could be? What is the smallest it could be?