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Advances on the Hamiltonian Problem – A Survey

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Abstract. This article is intended as a survey, updating earlier surveys in the area. For completeness of the presentation of both particular questions and the general area, it also contains material on closely related topics such as traceable, pancyclic and hamiltonian-connected graphs and digraphs.

1. Introduction

A graph G is *hamiltonian* if it contains a spanning cycle. The hamiltonian problem is generally considered to be determining conditions under which a graph contains a spanning cycle. Named for Sir William Rowan Hamilton, this problem traces its origins to the 1850s. Today, however, the constant stream of results in this area continues to supply us with new and interesting theorems and still further questions.

To many, including myself, any path or cycle problem is really a part of this general area and it is difficult to separate many of these ideas. Thus, although I will concentrate on spanning cycles (the classic hamiltonian problem), other related results, both stronger and weaker, will be presented in order to provide you with a better picture of the overall theory and problems as they exist today.

In doing this I shall generally restrict my attention to work done since [137] appeared in 1991, as earlier hamiltonian and related surveys (see [28], [42], [31], [185], [242], [25], [194], [43], [9], [88] and [137]) provide ample background on previous work. Thus, I shall expect my reader to be somewhat familiar with this area already. Since this area is so vast, I shall certainly be unable to mention everything, but shall do my best to cover important topics. However, I will cover only a limited amount dealing with closure operations as the recent survey [63] provides an excellent view of developments in this area and I shall not attempt to cover hamiltonian digraph results.

Throughout this article we consider finite simple graphs $G = (V, E)$, unless otherwise indicated. We reserve n to denote the order ($|V|$) and q the size ($|E|$) of G. We use $\delta(G)$ and $\Delta(G)$ for the minimum and maximum degrees of G respectively, and let $N(x)$ and $N(S)$ denote the neighborhood of the vertex x and set

S respectively. Further, let $c(G)$ denote the circumference of G, that is, the length of a longest cycle, $g(G)$, the girth, that is the length of a shortest cycle and

$$\sigma_k(G) = \min \{\deg x_1 + \cdots + \deg x_k \mid x_1, \ldots, x_k \text{ are independent in } G\}.$$

Graphs satisfying lower bounds on σ_k with $k \geq 2$ will often be called Ore-type graphs, while if $k = 1$, Dirac-type graphs. If G contains no induced subgraph isomorphic to any graph in the set $F = \{H_1, \ldots, H_k\}$, we say G is F-free, or H_1-free if F contains only H_1. For terms not defined here see [68].

2. Extending the Classics

In this section I will concentrate on results that generalize, or extend in some way, well-known hamiltonian results. Several such directions either emerged or were greatly developed over the past decade and a number of intriguing conjectures were solved.

An interesting problem concerning powers of hamiltonian cycles proved to be very difficult and developed in stages until finally resolved. The kth power of a graph G is the graph obtained from G by inserting edges between vertices at a distance at most k in G. Part (a) of the following Conjecture is due to Pósa (see [101]) while part (b) is due to Seymour [221]. Both parts generalize the classic result of Dirac [92].

Conjecture 1. (a) *If* $\delta(G) \geq 2n/3$, *then G contains the square of a hamiltonian cycle.* (b) *If* $\delta(G) \geq \frac{kn}{k+1}$, *then G contains the kth power of a hamiltonian cycle.*

Pósa's conjecture dates to 1962, but it was many years before a series of real advances were made on this question. Seymour indicated the difficulties involved here by observing that the truth of his conjecture would imply the difficult Hajnal-Szemerédi Theorem [143], that if $\Delta(G) < r$, then G is r colorable such that the sizes of the color classes are all $\lfloor n/r \rfloor$ or $\lceil n/r \rceil$.

A flurry of work on the Pósa conjecture began when M. S. Jacobson (unpublished) showed that if $\delta(G) \geq 5n/6$, then the conjecture holds. Then, Faudree, Gould, Jacobson and Schelp [118] showed that $\delta(G) \geq (3/4 + \epsilon)n + c(\epsilon)$ suffices. They later improved this to $\delta(G) \geq 3n/4$ (again unpublished). Fan and Häggkvist [107] further lowered the bound to $\delta(G) \geq 5n/7$. Fan and Kierstead (manuscript) then improved the bound to $(17n + 9)/24$. Faudree, Gould and Jacobson (manuscript) decreased the bound to $7n/10$ and Fan and Kierstead [108] showed that the Pósa condition was nearly optimal when they showed that $(2/3 + \epsilon)n + c(\epsilon)$ suffices. They also showed that $\epsilon = 0$ suffices if we only seek the square of a hamiltonian path [110]. Kierstead and Quintana [171] showed the Pósa Conjecture holds on graphs with minimum degree 2n/3 that also contain a maximal 4-clique. Fan and Kierstead [109] also gave conditions for a graph to contain two edge disjoint square hamiltonian cycles.

Turning to the Seymour conjecture, in [118] it was shown that for any $\epsilon \geq 0$ and positive integer k, there is a constant C such that if a graph G satisfies $\delta(G) \geq ((2k-1)/(2k)+\epsilon)n + C$, then G contains the kth power of a hamiltonian cycle. In [172], the above was improved to $(\frac{k}{k+1}+\epsilon)n$.

Ultimately, in [173] and [174], the truth of both the Pósa and Seymour Conjectures were verified for large n by Komlós, Sáközy and Szemerédi. I combine these results below.

Theorem 1 [173], [174]. *There exists a natural number n_0 such that if G has order n and $n \geq n_0$* and $\delta(G) \geq kn/(k+1)$, *then G contains the k-th power of a hamiltonian cycle.*

The main tools used in proving these results are the well-known regularity lemma [230] and the powerful Blow-up Lemma [176]. The regularity lemma has long been recognized as one of the best tools for dealing with problems on dense graphs. Recently, it has been emerging as a very effective approach to difficult cycle results.

A related result, also of Dirac type, is due to Aigner and Brandt [4]. This one concerns subgraphs of maximum degree two, originally conjectured in a somewhat weaker form by Sauer and Spencer [216].

Theorem 2 [4]. *Every graph G of order n with $\delta(G) \geq (2n-1)/3$ contains any graph with at most n vertices and maximum degree two.*

Corollary 3 [4]. *Let G be a graph of order n with $\delta(G) \geq (2n-1)/3$ and suppose $n \geq n_1 + n_2 + \cdots + n_k$ where $n_i \geq 3$ for all i. Then G contains the vertex disjoint union of the cycles $C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_k}$.*

Clearly then, any such graph contains any 2-factor we would want and hence provides a strong analogue to Dirac's theorem. I should mention that Alon and Fischer [6] also provided a solution to the Sauer-Spencer conjecture ($\delta = 2n/3$). Their result used work dependent on the regularity lemma and thus holds only for large graphs.

Related to the last result is another old conjecture due to El-Zahar [95].

Conjecture 2. *Let G be a graph of order $n = n_1 + n_2 + \cdots + n_k$ with $\delta(G) \geq \sum_{i=1}^{k} \lceil n_i/2 \rceil$, then G contains the 2-factor $C_{n_1} \cup \ldots \cup C_{n_k}$.*

Note that the graph $K_{s-1} + K_{\lceil\frac{n-s+1}{2}\rceil, \lceil\frac{n-s+1}{2}\rceil}$ has minimum degree $(n+s-1)/2$ but contains no s vertex disjoint odd length cycles. Thus, the conjecture is best possible.

El-Zahar [95] provided an affirmative answer to the case $k = 2$, while Dirac's Theorem handles $k = 1$. Recently, Abbasi [1] announced a solution for large n using the regularity lemma. It would still be interesting to find a solution to this beautiful conjecture for all n. It should be noted that Corrádi and Hajnal [87] provided an affirmative answer to the El-Zahar conjecture for the case that each $n_i = 3$.

An old conjecture of Erdős and Faudree [102] generalizes the Corrádi-Hajnal theorem in another direction.

Conjecture 3. *Let G be a graph with order $n = 4k$ and $\delta(G) \geq 2k$, then G contains k vertex disjoint 4-cycles.*

Alon and Yuster [8] proved that for any $\epsilon > 0$, there exists k_0 such that if G has order $4k$ and $\delta(G) \geq (2+\epsilon)k$ with $k \geq k_0$, then G contains k disjoint 4-cycles. In [207], a near solution was provided by Randerath, Schiermeyer and Wang.

Theorem 4. *Let G be a graph of order 4k and minimum degree at least 2k. Then G contains a vertex disjoint collection of subgraphs, $k-1$ of which are 4-cycles and the remaining subgraph has order 4 and at least four edges.*

Finally, in [175], a solution for large n was indicated as a consequence of another result (again dependent upon the regularity lemma).

Next we turn our attention to another generalization of the classic results of Dirac [92] and Ore [199]. Recall that a 2-factor is a 2-regular spanning subgraph of G. A hamiltonian cycle is then a 2-factor, and in one sense, it is the simplest 2-factor as it is composed of a single cycle. In another sense, it may be the most difficult 2-factor to find, as we must force a single cycle. We now ask the question: If we weaken the conditions that allow total control of the structure of a 2-factor (as was done above), can we still at least control the number of cycles in the 2-factor?

Theorem 5 [56]. *If G is a graph of order n satisfying*
(1) $\delta(G) \geq n/2$ and $n \geq 4k$ or
(2) $\sigma_2(G) \geq n$ and $n \geq 4k$
then G contains a 2-factor with exactly k cycles, and this result is best possible.

To see this result is best possible we need only consider the complete bipartite graph $K_{n/2,n/2}$. Clearly, the smallest cycle in any 2-factor of this graph is a 4-cycle and hence the bound on n is sharp. Further, $K_{(n-1)/2,(n+1)/2}$ (n odd), does not contain a 2-factor, hence the degree conditions are also sharp.

Theorem 5 was a natural direction to take, given the fact that Ore-type graphs are either pancyclic (contain cycles of all possible lengths) or $K_{n/2,n/2}$. Upon viewing Theorem 5, the next question is obvious.

Question 1. *Which of the conditions implying a graph G of order n is hamiltonian also imply that G contains a 2-factor with k cycles for all values of k, $1 \leq k \leq f(n)$. Further, in each case how large can $f(n)$ be?*

A number of results along these lines followed. I shall only mention a few.

Generalizing the result of Matthews and Sumner [192] on claw-free hamiltonian graphs, the following was shown in [72].

Theorem 6. *Let G be a 2-connected, claw-free graph of order* $n \geq 51$ with $\delta(G) \geq \frac{1}{3}(n-2)$. *Then for each k with* $1 \leq k \leq \frac{n-24}{3}$, *G has a 2-factor with exactly k cycles.*

The next result is from [71] and concerns a bipartite version of the question.

Theorem 7. *Let k be a positive integer and let G be a balanced bipartite graph of order* $2n$ *where* $n \geq \max\{51, \frac{k^2}{2}+1\}$. *If* $\deg u + \deg v \geq n+1$ *for every* $u \in V_1$ *and* $v \in V_2$, *then G contains a 2-factor with exactly k cycles.*

Extending Jung's [165] result on 1-tough graphs (restricted to minimum a degree condition), the following was shown in [117].

Theorem 8. *If G is a* 1*-tough graph of sufficiently large order n with* $\delta(G) \geq \frac{n-t}{2}$ $(0 \leq t \leq 4)$, *then G contains a* 2*-factor with k cycles where* $1 \leq k \leq \frac{n}{4} - t$.

Recall, a *dominating circuit* of a graph G is a circuit of G with the property that every edge of G either belongs to the circuit or is adjacent to an edge of the circuit. The next classic result is by Harary and Nash-Williams [144].

Theorem 9. *Let G be a graph without isolated vertices. Then* $L(G)$ *is hamiltonian if and only if* $G \simeq K_{1,n}$, *for some* $n \geq 3$, *or G contains a dominating circuit. An old conjecture of*

In order to generalize Theorem 9 we say that G contains a *k-system that dominates* if G contains a collection of k edge disjoint circuits and stars, (here stars are K_{1,n_i}, $n_i \geq 3$), such that each edge of G is either contained in one of the circuits or stars, or is adjacent to one of the circuits. With this in mind, the following was shown in [140].

Theorem 10. *Let G be a graph with no isolated vertices. The graph* $L(G)$ *contains a* 2*-factor with k* $(k \geq 1)$ *cycles if, and only if, G contains a k-system that dominates.*

Using Theorem 10, Hynds [153] investigated Question 1 in various graphs whose line graphs were already known to be hamiltonian.

Now we turn to another strong hamiltonian property introduced by Chartrand (see [198]). A graph is *k-ordered* (*hamiltonian*) if for every ordered sequence of k vertices there is a (hamiltonian) cycle that encounters the vertices of the sequence in the given order. Clearly, every hamiltonian graph is 3-ordered hamiltonian.

Ng and Schultz [198] were the first to investigate such graphs.

Theorem 11 [198]. *Let G be a graph of order n and let k be an integer with* $3 \leq k \leq n$. *If* $\deg(u) + \deg(v) \geq n + 2k - 6$ *for every pair* u, v *of nonadjacent vertices of G, then G is k-ordered hamiltonian.*

Corollary 12 [198]. *Let G be a graph of order n and let k be an integer with* $3 \leq k \leq n$. *If* $\deg(u) \geq n/2 + k - 3$ *for every vertex u of G, then G is k-ordered hamiltonian.*

Clearly, this theorem and corollary are analogs of Ore's [199] and Dirac's [92] fundamental results, respectively. Both bounds for k-ordered hamiltonicity were improved for small k with respect to n. Theorem 11 was improved by Faudree, Faudree, Gould, Jacobson and Lesniak [114].

Theorem 13 [114]. *Let $k \geq 3$ be an integer and let G be a graph of order $n \geq 53k^2$. If $\deg(u) + \deg(v) \geq n + (3k-9)/2$ for every pair u, v of nonadjacent vertices of G, then G is k-ordered hamiltonian.*

Corollary 12 was improved by Kierstead, Sárközy and Selkow [170] as follows.

Theorem 14 [170]. *Let $k \geq 2$ be an integer and let G be a graph of order $n \geq 11k - 3$. If $\deg(u) \geq \lceil \frac{n}{2} \rceil + \lfloor \frac{k}{2} \rfloor - 1$ for every vertex u of G, then G is k-ordered hamiltonian.*

We note that both of these bounds are sharp for the respective values of k. Unexpectedly, for small k, the Dirac type bound does not follow from the Ore type bound. In [119], this was further investigated and the following shown:

Theorem 15 [119]. *Let k be an integer with $3 \leq k \leq n/2$ and let G be a graph of order n. If $\deg(u) + \deg(v) \geq n + (3k-9)/2$ for every pair u, v of nonadjacent vertices of G, then G is k-ordered hamiltonian.*

The bound in Theorem 15 is sharp and for large k it implies the bound of Dirac-type. Thus,

(a) for large k, the Ore type bound yields the Dirac type bound;
(b) for small k, the Ore type bound is more than twice the Dirac type bound; and
(c) for moderate k, the situation is still not clear.

We summarize the above more precisely as follows. Let $\delta(n,k)$ be the smallest integer m for which any graph of order n with minimum degree at least m is k-ordered hamiltonian. The following theorem is from [119].

Theorem 16. *For positive integers k,n with $3 \leq k \leq n$ we have*

(i) $\delta(n,k) = \lceil \frac{n}{2} \rceil + \lfloor \frac{k}{2} \rfloor - 1,$ *for* $k \leq (n+3)/11$;
(ii) $\delta(n,k) > \frac{n}{2} + \frac{k}{2} - 2,$ *for* $(n+3)/11 < k \leq n/3$;
(iii) $\delta(n,k) \geq 2k - 2,$ *for* $n/3 < k < 2(n+2)/5$;
(iv) $\delta(n,k) = \lceil n/2 + \frac{3k-9}{4} \rceil,$ *for* $2(n+2)/5 \leq k \leq n/2$;
(v) $\delta(n,k) = n - 2,$ *for* $n/2 < k \leq 2n/3$; *and*
(vi) $\delta(n,k) = n - 1,$ *for* $2n/3 < k \leq n$.

Ng and Schultz [198] showed that k-ordered graphs must be $(k-1)$-connected. The degree conditions of the above results are enough to accomplish this level of connectivity. It is natural to ask if strengthening the connectivity conditions would allow us to lower the degree conditions. In [73] this question was investigated.

Theorem 17 [73]. *Let G be a graph on n vertices with* $\sigma_2(G) \geq n$. *Let* $k \leq n/176$ *be an integer. If G is* $\lfloor 3k/2 \rfloor$*-connected, then G is k-ordered hamiltonian.*

The connectivity bound is best possible, as illustrated by the following graph G_2. Let L_2, M_2, R_2 be complete graphs with $|R_2| = \lfloor k/2 \rfloor$, $|M_2| = 2\lfloor k/2 \rfloor - 1$, $|L_2| = n - |M_2| - |R_2|$. Let G_2' be the union of these three graphs, adding all possible edges containing vertices of M_2. Let $x_i \in L_2$ if i is odd, and let $x_i \in R_2$ otherwise. Add all edges x_ix_j whenever $|i-j| \notin \{0, 1, k-1\}$, and the resulting graph is G_2. The degree sum condition is satisfied and G_2 is $(\lfloor 3k/2 \rfloor - 1)$-connected. But there is no cycle containing the x_i in the proper order, since such a cycle would contain $2\lfloor k/2 \rfloor$ paths through M_2.

A slight improvement is possible when considering only minimum degrees. Again, the connectivity bound is best possible.

Theorem 18 [73]. *Let G be a graph on n vertices with minimum degree* $\delta(G) \geq n/2$. *Let* $k \leq n/176$ *be an integer. If G is* $3\lfloor k/2 \rfloor$*-connected, then G is k-ordered hamiltonian.*

We note also that a consequence of a result of Bollobás and Thomason [39] implies that every $22k$-connected graph is k-ordered. Thus, connectivity alone will suffice. They naturally raise the following question.

Question 2. *What is the least connectivity* $f(k)$ *so that any* $f(k)$*-connected graph is k-ordered?*

A natural variation of k-ordering would be to consider ordered edge sets, and in fact, even more can be said. We say L is a (k, t, s)-*linear forest* if L is a sequence $L = P^1, P^2, \ldots, P^t$ $(1 \leq t \leq k)$ of t disjoint paths, s of them being singletons such that $|V(L)| = k$. A graph G is (k, t, s)-*ordered* if for every (k, t, s)-linear forest L in G, there exists a cycle C in G that contains the paths of L in the designated order. Further, if the paths of L are each oriented and C can be chosen to encounter the paths of L in the designated order and according to the designated orientation on each path, then we say G is *strongly* (k, t, s)-*ordered*. If C is a hamiltonian cycle then we say G is (k, t, s)-*ordered hamiltonian* and *strongly* (k, t, s)-*ordered hamiltonian*, respectively. Note that saying G is (s, s, s)-ordered is the same as saying G is s-ordered. The following two results were shown in [70].

Theorem 19 [70]. *For* $k \geq 1$ *and* $1 \leq t \leq k$, *if G is a (strongly)* (k, t, s)*-ordered graph on* $n \geq k$ *vertices with* $\delta(G) \geq \frac{n+k-t+s}{2}$, *then G is (strongly)* (k, t, s)*-ordered hamiltonian.*

Theorem 20 [70]. *If* $s = t = k \geq 3$ *or* $0 \leq s < t < k$, *and G is a graph of order* $n \geq \max\{178t + k, 8t^2 + k\}$ *with*

$$\sigma_2(G) \geq \begin{cases} n + k - 3 & \text{if } s = 0 \\ n + k + s - 4 & \text{if } 0 < 2s \leq t \\ n + k + \frac{t-9}{2} & \text{if } 2s > t \end{cases};$$

then G is strongly (k, t, s)*-ordered.*

Finally, Ellingham, Zha and Zhang [94] took a different approach. Define a 2-trail as a trail that uses every vertex at most twice. Thus, spanning 2-trails generalize hamiltonian paths and cycles. They prove the following three results, each of which is sharp.

Theorem 21. (1) *If $\sigma_3(G) \geq n-1$, then G has a spanning 2-trail, unless $G = K_{1,3}$.*

(2) *If $\sigma_3(G) \geq n$, then either G has a hamiltonian path or a closed spanning 2-trail.*

(3) *If G is 2-edge connected and $\sigma_3(G) \geq n$, then G has a closed spanning 2-trail, unless $G = K_{2,3}$ or $K^*_{2,3}$ (the 6 vertex graph obtained from $K_{2,3}$ by subdividing one edge).*

3. Density

By density we mean conditions that force the existence of a sufficient number of edges to imply the desired result. Since most hamiltonian results are of this type, I shall restrict attention in this section to results involving size, degrees, or neighborhood conditions. Degree conditions are the classic approach to hamiltonian problems and neighborhood unions are a form of generalized degree condition. Early results of this type are discussed in a number of the previously mentioned surveys.

We shall consider several strengthenings of classic results as well as density conditions that imply new strong hamiltonian properties or generalizations of old properties. The first such generalization we consider is due to Brandt [53]. He defines a graph to be *weakly pancyclic* if it contains cycles of every length from the girth to the circumference. Brandt then showed the following.

Theorem 22. *If G is a nonbipartite graph of order n and size $q > \lfloor (n-1)^2/4 + 1 \rfloor$, then G is weakly pancyclic.*

Note that such graphs contain triangles. Brandt, Faudree and Goddard [55] then considered degree conditions for weakly pancyclic graphs.

Theorem 23 [55]. (a) *Let G be a 2-connected nonbipartite graph with $\delta(G) \geq n/4 + 250$. Then G is weakly pancyclic unless G has odd girth 7, in which case it has every cycle from 4 up to its circumference except for the 5-cycle.*

(b) *Every nonbipartite graph with $\delta(G) \geq (n+2)/3$ is weakly pancyclic (and has girth 3or 4).*

Brandt [52] also considered other degree conditions for weakly pancyclic graphs.

Theorem 24. *Let $G \neq C_5$ be a nonbipartite triangle-free graph of order n. If $\delta(G) > n/3$, then G is weakly pancyclic with girth 4 and circumference $\min\{2(n - \alpha(G)), n\}$, (where $\alpha(G)$ is the independence number of G).*

Brandt [53] also conjectured the following.

Conjecture 4. *Every nonbipartite graph of order n and size at least* $(n-1)(n-3)/4+4$ *is weakly pancyclic.*

Bollobás and Thomason [40] came very close to solving this conjecture. In fact, their work actually shows that a minimal counterexample to Brandt's Conjecture has small order (≤ 132).

Theorem 25 [40]. *Let G be a nonbipartite graph of order n and size at least* $\lfloor n^2/4 \rfloor - n + 59$. *Then G contains a cycle of length* ℓ for $4 \leq \ell \leq c(G)$.

Xiong [243] also considered weakly pancyclic line graphs (although using the term subpancyclic) satisfying a degree condition for adjacent pairs.

In a very different strengthening of Dirac's Theorem, Kaneko and Yoshimoto [167] show small subsets of vertices can be distributed along a hamiltonian cycle. Here $dist_C$ means the distance along the cycle C.

Theorem 26. *Let G be a graph of order n with* $\delta(G) \geq n/2$ *and let d be a positive integer with* $d \leq n/4$. *Then for any vertex subset A with at most* $n/2d$ *vertices, there exists a hamiltonian cycle C with* $dist_C(u, v) \geq d$ *for any* $u, v \in A$.

This result is sharp in that the bound on the cardinality of A cannot be increased. With this interesting result in hand, we raise a natural problem.

Problem 1. *What other density conditions allow the distribution of "small" sets of vertices along a hamiltonian cycle? Are there density conditions that do not allow such a distribution, except possibly on a constant number of vertices?*

Another generalization of hamiltonian graphs is the idea of cyclable sets. A subset S of $V(G)$ is called *cyclable* in G if all the vertices of S belong to a common cycle in G. Clearly, if $V(G)$ is cyclable, then G is hamiltonian. Also, if G is hamiltonian, then S is cyclable for any $S \subset V(G)$. If S is a subset of $V(G)$, we let $G[S]$ denote the subgraph induced by S and $\alpha(S, G)$ be the independence number of $G[S]$. A number of set restricted density results imply cyclability. The first extends the well-known Chvátal-Erdős Theorem [80].

Theorem 27 [130]. *Let G be a k-connected graph* $(k \geq 2)$ *and S a subset of* $V(G)$. *If* $\alpha(S, G) \leq k$ *then S is cyclable in G.*

The next result is due independently to Bollobás and Brightwell [33] (as a corollary to a more general result) and Shi [226] (stated as a lemma). It uses the classic Dirac-type density condition for the subset S of $V(G)$. Let $\delta(S, G)$ be the minimum degree in G of a vertex of S.

Theorem 28. *Let G be a* 2*-connected graph and S a subset of* $V(G)$. *If* $\delta(S, G) \geq n/2$ *then S is cyclable in G.*

Ota [201] made the natural extension to degree sums of pairs of nonadjacent vertices in S, denoted $\sigma_2(S,G)$.

Theorem 29. *Let G be a 2-connected graph of order n and S a subset of $V(G)$.* If $\sigma_2(S,G) \geq n$, *then S is cyclable in G.*

This was further pushed to sums of three vertices in [127], extending an earlier result of Flandrin, Jung and Li [126].

Theorem 30 [127]. *Let G be a 2-connected graph of order n and S a subset of $V(G)$. If $\deg x + \deg y + \deg z \geq n + |N(x) \cap N(y) \cap N(z)|$ for any three independent vertices $x, y, z \in S$, then S is cyclable in G.*

The next result of Broersma, Li, Li, Tian and Veldman [61] extends the hamiltonian work in [20]. Here $\kappa(S,G)$ is the minimum cardinality of a set of vertices separating two vertices of S and $\sigma_t(S,G)$ is the degree sum in G of any t nonadjacent vertices of S.

Theorem 31. *Let G be a 2-connected graph of order n and S a subset of $V(G)$. If $\sigma_3(S,G) \geq n + \min\{\kappa(S,G), \delta(S,G)\}$, then S is cyclable in G.*

While in [145] 3-connected graphs were studied.

Theorem 32. *Let G be a 3-connected graph of order n and S a subset of $V(G)$.*
(a) *If $\sigma_4(S,G) \geq n + 2\alpha(S,G) - 2$, then S is cyclable.*
(b) *If $\sigma_4(S,G) \geq n + \delta(S,G)$ and $\deg v \geq n/2$ for every $v \in S - N[w]$, where $w \in S$ and $\deg w = \delta(S,G)$, then S is cyclable in G.*
(c) *If $\sigma_2(G) \geq n/2 + \delta(G)$, then G is hamiltonian.*

Recently, a bipartite version for cyclable sets was also found.

Theorem 33 [2]. *Let $G = (X \cup Y, E)$ be a 2-connected balanced bipartite graph of order $2n$ and S is a subset of X. If $\deg x + \deg y \geq n + 1$ for every nonadjacent pair $x \in S$, $y \in Y$, then S is cyclable in G.*

Polický [206] defined $\omega(u,v)$ as the number of components of $G[N(u)]$ containing no neighbor of v. He then proved the following result.

Theorem 34. *In a graph G of order n, if $\deg u + \deg v + \max\{\omega(u,v), \omega(v,u)\} \geq n$ for each pair of nonadjacent vertices u and v, then G is hamiltonian.*

Stacho [217] gave a sufficient condition of degree sum type for a graph to be hamiltonian. This condition generalizes several old results. The condition is: Let G be a connected graph with degree sequence $d_1 \leq d_2 \leq \cdots \leq d_n$. Suppose that whenever $i \leq n/2$, $i \neq j$, $d_i \leq i$ and $d_j \leq j - 1$ all hold, then at least one of four properties for the pair (d_i, d_j) also holds. The four conditions are based on Polický's parameter for the number of components not containing a neighbor of a vertex v. They are technical and not presented here.

Stacho further exploited the Polický parameter to study cycles through specified vertices [219] and a closure type result for long cycles [218].

In [98], Enomoto, Kaneko and Tuza conjectured that if $\sigma_k(G) \geq n$ or $\alpha(G) < k$, then $V(G)$ can be covered by $k-1$ cycles, edges or vertices. Note that when $k=2$ this conjecture is answered by Ore's Theorem [199]. The case $k=3$ was shown by Enomoto, Kaneko, Kouider and Tuza [97]. In [179], this conjectured was settled in general.

Theorem 35. *Let G be a graph of order n and let $X \subset V(G)$. If $\sigma_k(X, G) \geq n$ or $\alpha(X, G) < k$, then X can be covered with $k-1$ cycles, edges or vertices.*

It is worth mentioning that a weaker statement: If $\delta(G) \geq n/k$, then G can be covered by $k-1$ cycles, edges or vertices, was previously considered. It clearly generalizes Dirac's Theorem for $k=2$. The case $k=3$ was shown by Enomoto, Kaneko and Tuza [98] and for every $k \geq 2$ by Kouider [178].

Bondy [41] showed that all graphs satisfying Ore's condition are either pancyclic or isomorphic to $K_{n/2,n/2}$. Aldred, Holton and Min [5] relaxed Ore's condition by considering graphs with $\sigma_2 \geq n-1$.

Theorem 36. *If G satisfies $\sigma_2(G) \geq n-1$, then G is pancyclic unless G is isomorphic to one of the following graphs:*
(a) *a graph of order n consisting of two complete graphs joined at a vertex,*
(b) *a subgraph of the join of a complete graph of order $(n-1)/2$ and an empty graph of order $(n+1)/2$,*
(c) $K_{n/2,n/2}$,
(d) C_5.

Brandt and Veldman [58] considered $\deg x + \deg y \geq n$ for every adjacent pair x, y in G. They showed that if G satisfies the degree condition on edges then the circumference of G is precisely $n - s(G)$, where $s(G) = \max\{0, \max_S(|S| - |N(S)| + 1)\}$, where the inner max ranges over all nonempty sets S of independent vertices of G with $S \cup N(S) \neq V(G)$.

A graph is *pancyclic modulo k* if it contains cycles of all lengths modulo k. Dean [89] asked the following question:

Question 3. *Which graphs are pancyclic modulo k?*

Dean [89] showed that every 3-connected planar graph (except K_4) with minimum degree at least k is pancyclic modulo k.

Density conditions in G certainly effect the line graph $L(G)$. In [239], van Blanken, van den Heuvel, and Veldman considered $f(n)$ as the smallest integer such that for every graph G of order n with minimum $\delta(G) > f(n)$, then $L(G)$ is pancyclic whenever $L(G)$ is hamiltonian. They were able to provide results showing that $f(n) = \Theta(n^{1/3})$.

Another interesting Ore-type result involving even more structure than a hamiltonian cycle was found by Mao-cheng, Li and Kano [190]. Here a $[k, k+1]$-factor means a factor where each vertex has degree k or $k+1$.

Theorem 37 [190]. *Let $k \geq 2$ be an integer and G a graph of order $n \geq 3$ with $\delta(G) \geq k$. Assume that $n \geq 8k - 16$ for even n and $n \geq 6k - 13$ for odd n. If $\sigma_2(G) \geq n$, then for any given hamiltonian cycle C, G has a $[k, k+1]$-factor containing C.*

This result extents a similar Dirac-type result in [241].

Turning to regular graphs, a number of results appeared attempting to strengthen or generalize Jackson's classic result [155] that every 2-connected k-regular graph on at most $3k$ vertices is hamiltonian.

Theorem 38 [147]. *Let G be a 2-connected k regular graph on at most $3k + 3$ vertices, Then G is hamiltonian or G is the Petersen graph or the Petersen graph with one vertex replaced by a triangle.*

The following conjecture would improve Jackson's Theorem for 3-connected graphs.

Conjecture 5 [157]. *For $k \geq 4$, every 3-connected k-regular graph on at most $4k$ vertices is hamiltonian.*

This conjecture is a special case of a conjecture of Häggkvist (see [157]) which was shown not to hold in general. A number of others have considered the question of dominating cycles in regular graphs. For more information on this see [59].

Turning to neighborhood conditions, I would advise the reader new to these conditions to begin with [185] and [137]. I will try not to repeat earlier results mentioned in these two papers.

In [120] independence number is tied to neighborhood union conditions. Here we let $\delta_k(G) = \min|\cup_{u \in S} N(u)|$, where the minimum is taken over all k element subsets S of $V(G)$.

Theorem 39. *Let $r \geq 1$ and $m \geq 3$ be integers. Then for each nonnegative function $f(r, m)$ there exists a constant $C = C(r, m, f)$ such that if G is a graph of order n $(n > r, n > m)$ with $\delta_r(G) \geq n/3 + C$ and at most $f(r, m)$ independent vertices, then*
(a) *G is traceable if $\delta_1(G) \geq r$ and G is connected;*
(b) *G is hamiltonian if $\delta_1(G) \geq r + 1$ and G is 2-connected;*
(c) *G is hamiltonian-connected if $\delta_1(G) \geq r + 2$ and G is 3-connected.*

Similar results are also shown for claw-free graphs in [120].

Song [227] considered a Fan-like neighborhood condition.

Theorem 40. *Let G be a 2-connected graph of order $n \geq 3$ with connectivity k. If there exists an integer t such that for any distinct vertices u and v, $dist(u, v) = 2$ implies that $|N(u) \cup N(v)| \geq n - t$, and for any independent set S of cardinality $k + 1$ we have that $\max\{\deg u \mid u \in S\} \geq t$, then G is hamiltonian.*

Broersma, van den Heuvel and Veldman [64] sharpened earlier results with the following theorem.

Theorem 41. *If G is a 2-connected graph of order n such that $|N(u) \cup N(v)| \geq n/2$ for every pair of nonadjacent vertices u and v, then either G is hamiltonian, the Petersen graph, or belongs to one of three families of exceptional graphs with connectivity 2 (see Figure 1).*

Clearly a consequence of the last theorem is that if G is 3-connected and satisfies the neighborhood condition, then G is hamiltonian or the Petersen graph. This result verifies a conjecture of Jackson [156] concerning 2-connected graphs and a conjecture of Chen concerning 3-connected graphs.

Chen and Schelp [77] extended many known results with the following idea. A sequence of real numbers $c_1, \ldots, c_{k+1}$ is called an Hk-sequence if $c_1|S_1| + c_2|S_2| + \cdots + c_{k+1}|S_{k+1}| > n-1$ for any independent set S of order $k+1$, where $S_i = \{v \in V(G) | \ |N(v) \cap S| = i\}$.

Theorem 42. *A sequence $c_1 \leq 1, c_2, \ldots, c_k, c_{k+1} \leq 2$ is an Hk sequence if the following two conditions are satisfied.*
(1) *For each collection of indices $i_1, i_2, \ldots, i_l, \ldots$ (allowing repetitions)*

$$\sum_{i_l} (i_l - 1) \leq k - 1 \text{ implies } \sum_{i_l} (c_{i_l} - 1) \leq 1.$$

(2) $c_i + 2c_{k+2-i} \leq 5$ *for* $2 \leq i \leq k-1$.

Subsequently, Ainouche and Schiermeyer [12] further generalized this work. For an independent set $S \subset V(G)$ of $t+1$ vertices define the $t+1$ neighborhood intersections $S_i = \{\ v \in V(G) - S \mid |N(v) \cap S| = i\}$, $1 \leq i \leq t+1$. Let $|S_i| = s_i$.

Theorem 43 [12]. *Let G be a 2-connected graph of order n. Then G is hamiltonian or there exists an independent set $X \subset V(G)$ of cardinality $t+1$, $1 \leq t \leq \kappa(G)$ such that*

$$\sum_{i=1}^{t+1} w_i s_i \leq n - 1 - \sum_{j>2} |N_j(X)|$$

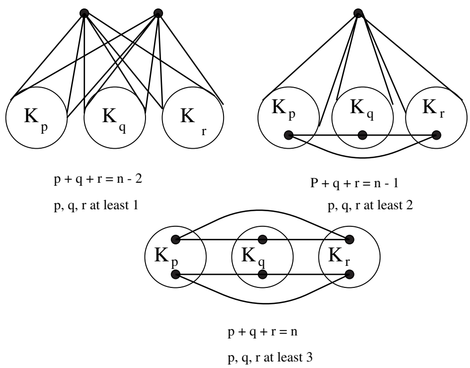

Fig. 1. Three exceptional families of graphs

where w_i, $1 \le i \le t+1$ *are real numbers satisfying* $0 \le w_1 \le 1$, *and for* $1 < i_1 \le i_2 \le \ldots \le i_m \le t+1$ *and* $\sum_{j=1}^{m} i_j \le t+1$ *we have* $\sum_{j=1}^{m} (w_{i_j} - 1) \le 1$; *where* $N_j(X)$ *denotes the set of vertices whose nearest vertex in* X *is at distance* j.

Song and Zhang [228] also improved several known results with the following stronger theorem.

Theorem 44. *Let* G *be a graph of order* $n \ge 3$ *with connectivity* $k \ge 2$ *and independence number* α. *Let every independent set* S *of* $k+1$ *vertices satisfy one of the following:*
(1) *there exists* $u \ne v$ *in* S *such that* $\deg u + \deg v > n$ *or* $|N(u) \cap N(v)| \ge \alpha$;
(2) *for any pair* $u \ne v$ *in* S, $|N(u) \cup N(v)| \ge n - \Delta(S)$;
then G *is hamiltonian.*

Chen and Liu [76] considered arbitrary independent k-sets for their neighborhood unions.

Theorem 45. *Let* $k \ge 1$ *be a fixed integer. In a* $(4k-4)$*-connected graph* G *of order* $n \ge 3$, *if* $|N(S)| + |N(T)| \ge n$ *for every two disjoint independent sets* S *and* T *of* k *vertices*, *then* G *is hamiltonian.*

In a different direction, Faudree and van den Heuvel [125] showed a weakening of the classic Ore-type condition, along with a new structure assumption was possible.

Theorem 46. *Let* G *be a* 2*-connected graph of order* n *with* $\sigma_2(G) \ge n-k$ *and suppose that* G *has a* k*-factor. Then* G *is hamiltonian.*

Finally, Chen and Jacobson [74] provided an improved degree condition in k-partite graphs.

Theorem 47. *If* G *is a balanced* k*-partite graph of order* kn *such that for each pair of nonadjacent vertices* x, y *in different parts*, $\deg x + \deg y > \left(k - \frac{2}{k+1}\right)n$ *for* k *odd and* $\deg x + \deg y > \left(k - \frac{4}{k+2}\right)n$ *for* k *even, then* G *is hamiltonian.*

4. More Than One Hamiltonian Cycle

The fundamental question that dominates this section is: When does a graph contain more than one hamiltonian cycle? A natural extension of this type of question is determining how many different cycles are possible. Another natural question is: When can the edge set of a graph be decomposed into disjoint hamiltonian cycles? We begin with the first question.

A classic result of Smith (see [238]) says that every edge of a 3-regular graph is contained in an even number of hamiltonian cycles. Thus, every 3-regular hamiltonian graph contains a second (and a third) hamiltonian cycle. Thomason [231] extended Smith's result to all r-regular graphs where r is odd (in fact, to all graphs in which all vertices have odd degree).

Sheehan [223] conjectured that every hamiltonian 4-regular graph has a second hamiltonian cycle. Since every r-regular graph (r even) is the union of pairwise edge-disjoint spanning 2-regular graphs, Sheehan's conjecture combined with the results of Smith and Thomason implies that every hamiltonian regular graph, except the cycle, has a second hamiltonian cycle. Thomassen [233] added the last piece of the puzzle when m is sufficiently large.

Theorem 48 [233]. *If G is hamiltonian and m-regular with $m \geq 300$, then G has a second hamiltonian cycle.*

Thomassen's proof uses a version of the Lovász Local Lemma [104] and is related to his proof of another cycle result in [234]. Using a Theorem of Fleischner and Stiebitz [129], Thomassen [234] verified that every longest cycle in a 3-connected, 3-regular graph has a chord. Thomassen [234] also provided the following general sufficient condition for the existence of a second hamiltonian cycle.

Theorem 49. *Let G be a graph with a hamiltonian cycle C. Let A be a vertex set in G such that A contains no two consecutive vertices of C and A is dominating in $G - E(C)$, then G has a hamiltonian cycle C' such that $C' - A = C - A$ and there is a vertex v in A such that one of the two edges of C' incident with v is in C and the other is not in C.*

Refining Thomassen's method, Horak and Stacho [152] obtained the following extension.

Theorem 50. *For any real number $k \geq 1$, there exists $f(k)$ so that every hamiltonian graph G with $\Delta(G) \geq f(k)$ has at least $\delta(G) - \lfloor\frac{\Delta(G)}{k}\rfloor + 2$ hamiltonian cycles. In particular, every hamiltonian graph with $\Delta(G) \geq f(\Delta(G)/\delta(G))$ has a second hamiltonian cycle.*

A graph is uniquely hamiltonian if it contains exactly one hamiltonian cycle. A question related to Sheehan's is the following:

Question 4. *Does every uniquely hamiltonian graph have a vertex of low degree?*

Entringer and Swart [100] constructed an infinite family of uniquely hamiltonian graphs with minimum degree three. However, it is not known if there exists a uniquely hamiltonian graph of minimum degree four [see [158]]. Jackson and Whitty [158] also showed that any uniquely hamiltonian graph contains a vertex of degree at most $(n + 9)/4$ and if there is a unique 2-factor, then the graph contains a vertex of degree 2. Bondy and Jackson [48] provided the best bound to date.

Theorem 51. *Every uniquely hamiltonian graph on n vertices has a vertex of degree at most $c \log_2 (8n) + 3$ where $c = (2 - \log_2 3)^{-1} \approx 2.41$.*

They further showed that every uniquely hamiltonian plane graph has at least two vertices of degree less than four and conjecture the following.

Conjecture 6. *Every uniquely hamiltonian planar graph has at least two vertices of degree two.*

The hunt for additional hamiltonian cycles is certainly not a new pursuit. In 1957 Kotzig (see [50]) asked which 4-regular, 4-connected graphs have a decomposition into two hamiltonian cycles. Independently, in 1971, Nash-Williams [196] asked if every 4-regular, 4-connected graph has a hamiltonian cycle and if in fact, it has two edge-disjoint hamiltonian cycles.

In 1956, Tutte [237] showed every 4-connected planar graph is hamiltonian. Martin [191] and independently Grunbaum and Malkevitch [142] showed 4-regular, 4-connected planar graphs need not have two edge disjoint hamiltonian cycles. Grunbaum and Malkevitch further asked if every 5-connected planar graph has two edge disjoint hamiltonian cycles. Zaks [244] and Rosenfeld [211] provided constructions yielding infinitely many examples of 5-connected planar graphs (both regular and nonregular) in which every pair of hamiltonian cycles have common edges.

Thomassen [235] considered the question of the number of hamiltonian cycles in bipartite graphs. He once again applied the techniques of Thomason [231].

Theorem 52 [235]. *Let $C : x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_1$ be a hamiltonian cycle in a bipartite graph G.*

(a) *If all the vertices $y_1, \ldots, y_n$ have degree at least 3, then G has another hamiltonian cycle containing the edge x_1y_1.*
(b) *If all the vertices $y_1, \ldots, y_n$ have degree $d > 3$ and if $P_1, P_2, \ldots, P_q$ $(0 \leq q \leq d-3)$ are paths in C of length 2 of the form $y_{i-1}x_iy_i$, then G has at least $2^{q+1-d}(d-q)!$ hamiltonian cycles containing $P_1 \cup \ldots \cup P_q$.*

Thomassen [235] also considered bipartite graphs of large girth. The following is a counterpart to part (b) above.

Theorem 53. *Let G be a bipartite graph of girth g and let $C : x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_1$ be a hamiltonian cycle in G. Assume that each vertex y_j $(1 \leq j \leq n)$ has degree 4. Let $P_1, P_2, \ldots, P_q$ be a (possibly empty) collection of paths each of the form $y_{i-1}x_iy_i$. Let E be the set of edges in G each joining two vertices of $P_1 \cup \ldots \cup P_q$ which have degree 2 in $P_1 \cup \ldots \cup P_q$. Let c denote the number of components of $P_1 \cup \ldots \cup P_q \cup E$, put $k = |V(P_1) \cup \ldots \cup V(P_q)|$. If $g \geq k + 5c + 1$, then the number of hamiltonian cycles in G which contain $P_1 \cup \ldots \cup P_q$ is at least $(3/2)^{(g-k-5c)/8}$.*

Thomassen [235] also posed a number of interesting problems. The first of these centers on a reduction method for hamiltonian graphs. We denote by G/e the graph obtained from G upon contracting the edge e.

Problem 2 [235]. *Does every hamiltonian graph G of minimum degree at least 3 contain an edge e such that $G - e$ and G/e are both hamiltonian?*

If G has two distinct hamiltonian cycles and e is an edge which belongs to one but not both, then e satisfies the question. Thus, Theorem 52(a) gives an affirmative answer to the problem for bipartite graphs. An affirmative answer for all graphs would prove the following conjecture by Thomassen [235]. Let $P(G, \lambda)$ denote the chromatic polynomial of the graph G.

Conjecture 7 [235]. *If G is a hamiltonian graph with n vertices then* $(-1)^n P(G, \lambda)$ *is positive for* $1 < \lambda < 2$.

Note that Theorem 52(b) gives a lower bound of $2^{1-d}d!$ on the number of hamiltonian cycles in a bipartite graph where all vertices of one color class have degree at least d. That this bound cannot be replaced by $(d!)^2$ can be seen by taking a cycle $x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_1$ and adding all edges of the form $x_i y_j$. Note that n may be arbitrarily large here. This leads to the problem:

Problem 3 [235]. *Does there exist a* 4*-regular bipartite hamiltonian graph with more than* 10^{10} *vertices and less than* 100 *hamiltonian cycles?*

The above question is answered affirmatively if 4-regular is replaced by 3-regular, thus leading one to suspect an affirmative answer.

Conjecture 8 [235]. *There exists a function* $f(g)$ *tending to infinity as g tends to infinity such that every bipartite hamiltonian graph of minimum degree* 3 *and girth g has at least* $f(g)$ *hamiltonian cycles.*

Finally, sufficiently strong conditions on minimum degree or girth may allow the above bipartite results to generalize to nonbipartite graphs.

Problem 4 [235]. *Does there exist a graph of minimum degree* 10^{10} *with precisely one hamiltonian cycle?*

We do not yet even know if there exist graphs of minimum degree 4 with precisely one hamiltonian cycle.

All the above problems and conjectures deal with hamiltonian graphs. This condition is not easily dropped as large girth in 3-connected cubic graphs need not imply the graph is hamiltonian. But perhaps large cyclic connectivity is sufficient. (The cyclic connectivity of a graph is the smallest number of edges that must be deleted in order to obtain a graph with at least two components containing cycles.)

Problem 5 [235]. *Does there exist a natural number m such that every cubic graph of cyclic connectivity at least m is hamiltonian?*

Density conditions have also been used to obtain results on multiple edge-disjoint hamiltonian cycles. Faudree, Rousseau and Schelp [123] gave an Ore-type condition for the existence of multiple edge disjoint hamiltonian cycles.

Theorem 54. *Let k be a positive integer.*

(a) *If G is a graph of order* $n \geq 60k^2$ *such that* $\sigma_2(G) \geq n + 2k - 2$, *then G contains k edge disjoint hamiltonian cycles.*

(b) *If G has order* $n \geq 6k$ *and size at least* $\binom{n-1}{2} + 2k$, *then G contains k edge disjoint hamiltonian cycles.*

Li and Zhu [189] also considered an Ore-type bound.

Theorem 55. *If G is a graph of order* $n \geq 20$ *with* $\delta(G) \geq 5$ *and* $\sigma_2(G) \geq n$, *then G has at least two edge disjoint hamiltonian cycles.*

Li [187] earlier had shown a more general result on edge disjoint cycles.

Theorem 56. *Let k be a positive integer. If G is a graph of order* $n \geq 8k^2 - 5$ with $\sigma_2(G) \geq n$ *and* $2k+1 \leq \delta(G) \leq 2k+2$, *and if* $l_1, \dots, l_k$ *are integers with* $3 \leq l_1 \leq \cdots \leq l_k \leq n$, *then G contains edge disjoint cycles of lengths* $l_1, \dots, l_k$ *respectively. In particular, G contains k edge disjoint hamiltonian cycles.*

Egawa [93] greatly strengthened the earlier works on k edge disjoint hamiltonian cycles by providing a linear bound for Ore-type graphs.

Theorem 57. *Let* $n, k \geq 2$ *be integers with* $n \geq 44(k-1)$. *If G is a graph of order n with* $\sigma_2(G) \geq n$ *and* $\delta(G) \geq 4k-2$, *then G contains k edge disjoint hamiltonian cycles.*

Next, we say a graph is of Fan-type $2k$ if the distance $dist_G(u, v) = 2$ implies that $\max\{\deg u, \deg v\} \geq n/2 + 2k$, The motivation for this term is the result of Fan[105], that says a 2-connected Fan-type 0 graph is hamiltonian.

In this spirit, Zhou [248] gave a sufficient condition for G to contain two edge disjoint hamiltonian cycles.

Theorem 58 [248]. *Let G be a* 4*-connected graph of order n that is Fan-type* 2. *Then G contains two edge disjoint hamiltonian cycles.*

Zhou [248] also conjectured the following extension, recently proved by Li [186].

Theorem 59 [186]. *Every* $2(k+1)$*-connected Fan-type* $2k$ *graph has* $k+1$ *edge disjoint hamiltonian cycles.*

Still open would be the question(s) of decomposing Fan-type $2k$ graphs into other fixed cycle lengths, or the question of how many cycles may be present in a 2-factor of a Fan-type $2k$ graph.

Several other questions can now be considered. A natural one is that of hamiltonian decompositions, that is, partitioning the edge set of G in hamiltonian cycles (if G is $2d$-regular) or hamiltonian cycles and a perfect matching (if G is $(2d+1)$-regular).

Perhaps the most general conjecture in this area is due to Nash-Williams [195] (and strengthened by Jackson [154]). For early work in this general area see [9].

Conjecture 9. *Every k-regular graph on at most $2k+1$ vertices is hamiltonian decomposable.*

For questions of hamiltonian decomposition, various graph products have received considerable attention. The typical question is:

Question 5. *If G_1 and G_2 are hamiltonian decomposable, is the appropriate product of G_1 and G_2 also hamiltonian decomposable?*

Since various products are often known under different names, I shall define the products in question. Each product graph has vertex set $V(G_1) \times V(G_2)$.

The cartesian product $G = G_1 \times G_2$ has edge set

$$E(G) = \{(u_1, u_2)(v_1, v_2) \mid u_1 = v_1 \text{ and } u_2v_2 \in E(G_2) \text{ or } u_2 = v_2 \text{ and } u_1v_1 \in E(G_1)\}.$$

The direct product (or conjunction) $G = G_1 \cdot G_2$ has edge set

$$E(G) = \{(u_1, u_2)(v_1, v_2) \mid u_1v_1 \in E(G_1) \text{ and } u_2v_2 \in E(G_2)\}.$$

The strong product $G = G_1 \otimes G_2$ has edge set

$$E(G) = \{(u_1, u_2)(v_1, v_2) \mid u_1 = v_1 \text{ and } u_2v_2 \in G_2, or$$

$$u_2 = v_2 \text{ and } u_1v_1 \in E(G_1), \text{ or both } u_1v_1 \in E(G_1) \text{ and } u_2v_2 \in E(G_2)\}.$$

Finally, the lexicographic product (sometimes called composition, tensor or wreath product) $G = G_1[G_2]$ has edge set

$$E(G) = \{(u_1, u_2)(v_1, v_2) \mid u_1v_1 \in E(G_1), \text{ or } u_1 = v_1 \text{ and } u_2v_2 \in E(G_2)\}.$$

Notable advances on product decompositions include the following:

Theorem 60 [16]. *The lexicographic product of two hamiltonian decomposable graphs is hamiltonian decomposable.*

This following conjecture is due to Alspach, Bermond and Sotteau [9] and is suggested by Theorem 60.

Conjecture 10. *If D_1 and D_2 are hamiltonian decomposable digraphs, then the lexicographic product of D_1 and D_2 is hamiltonian decomposable in general.*

The phrase "in general" above is necessary in the case that $|V(D_2)| = 2$ where failure can occur (see [197]). Ng [197] considers this question in digraphs and shows that the lexicographic product of hamiltonian decomposable digraphs is hamiltonian decomposable when the first digraph has odd order and the second has at least 3 vertices.

In 1978, Bermond [29] conjectured that the set of hamiltonian decomposable graphs is closed under cartesian product. Although this conjecture is not completely settled, the following result of Stong [229] makes a major contribution.

Theorem 61. *Let G_1 and G_2 be two graphs that are decomposable into s and t hamiltonian cycles, respectively, with $t \leq s$. Then $G_1 \times G_2$ is hamiltonian decomposable if one of the following holds:*
(1) $s \leq 3t$
(2) $t \geq 3$
(3) *the order of G_2 is even, or*
(4) *the order of G_1 is at least $6\lceil s/t \rceil - 3$.*

It is easy to see that if G_1 and G_2 are both bipartite, then the direct product $G_1 \cdot G_2$ is disconnected. Hence, the set of hamiltonian decomposable graphs is not closed under the direct product. Bosak [49] and Zhou [248] independently provided a case in which it is closed.

Theorem 62. *Suppose both G_1 and G_2 are hamiltonian decomposable. If at least one of them has odd order, then $G_1 \cdot G_2$ is hamiltonian decomposable.*

We say G is k-regularizable if multiple edges can be added to G (if necessary) to make the resulting multigraph G^* k-regular. Further, if at most one edge is added between any pair of vertices, we say G is k^*-regularizable, that is, provided the resulting multigraph G^* has at most two edges between any pair of vertices. Let H be a 4^*-regularizable connected spanning subgraph of a graph G. Then H^* is eulerian and is said to be a UOET graph if it admits an eulerian tour in which no proper closed subtrail is of even length. Using these ideas Balakrishnan and Paulraja [13] characterized those graphs G such that $G \cdot K_2$ has a hamiltonian cycle.

Theorem 63 [13]. *For any graph G, $G \cdot K_2$ is hamiltonian if and only if G has a UOET-subgraph.*

They also provided counterexamples to several conjectures of Jha [164]. These include: If G is a nonbipartite hamiltonian decomposable graph of even order, then $G \cdot K_2$ is hamiltonian decomposable, as well as then $G \cdot C_n$ is hamiltonian decomposable.

For strong products, Zhou [248] provided the following:

Theorem 64. *If both G_1 and G_2 are hamiltonian decomposable and at least one of them has odd order, then $G_1 \otimes G_2$ is hamiltonian decomposable.*

This was improved recently in [106] where Fan and Liu show:

Theorem 65. *The set of hamiltonian decomposable graphs is closed under strong products, that is, if G_1 and G_2 are hamiltonian decomposable, then so is $G_1 \otimes G_2$.*

Kriesell [181] considered the hamiltonian question for the lexicographic product of two graphs, where the graphs have broader conditions than those stated above. In particular, he showed the following:

Theorem 66. (a) *If G is* 1*-tough and contains a* 2*-factor and* $|E(H)| \geq 2$ *then* $G[H]$ *is hamiltonian.*
(b) *If G is* 2*-tough and* $|E(H)| \geq 2$*, then* $G[H]$ *is hamiltonian.*
(c) *If G is connected and 2k-regular and* $|V(H)| \geq k$*, then* $G[H]$ *is hamiltonian.*
(d) *If G is* $(2k+1)$*-regular and connected and G has a* 1*-factor and* $|V(H)| \geq k+1$*, then* $G[H]$ *is hamiltonian.*
(e) *If G is connected and vertex transitive of degree k and* $|V(H)| \geq k/2$*, then* G[H] *is hamiltonian.*
(f) *If G is connected and vertex transitive and* $|E(H)| \geq 2$*, then* $G[H]$ *is hamiltonian.*
(g) *If G is cubic and* 2*-edge connected and* $|V(H)| \geq 2$*, then* G[H] *is hamiltonian. If G is* 4*-regular and connected and* $|V(H)| \geq 2$*, then* G[H] *is hamiltonian.*

Kriesell's result suggests the following general problem.

Problem 6. *What natural graph properties of G and H are sufficient to imply that the product of G and H is hamiltonian.*

The *block-intersection graph* for the Steiner System (S, B) is the graph $G(S, B)$ with $V(G(S, B)) = B$ and where two vertices are joined by an edge if and only if the corresponding blocks in (S, B) have a common element. It has been shown that graphs for a variety of designs including Steiner triple systems are hamiltonian (see [10] and [151]). Pike [203] showed the block-intersection graph of a Steiner triple system of order $n \leq 15$ is hamiltonian decomposable. Further, he conjectures:

Conjecture 11 [203]. *If* (S, B) *is a Steiner triple system, then its block-intersection graph is hamiltonian decomposable.*

A number of other special case conjectures are also worthy of mention here. A graph O_k (called the odd graph) is a k-regular graph whose vertices are indexed by the $(k-1)$-subsets of a $(2k-1)$-set and two vertices are adjacent if, and only if, their indexing subsets are disjoint. For example, O_3 is the Petersen graph. Meredith and Lloyd [193] showed that O_4, O_5 and O_6 are hamiltonian decomposable. Further, they conjectured:

Conjecture 12. *The odd graphs* O_k *are hamiltonian decomposable for* $k \geq 4$.

The boolean graphs B_k are k-regular bipartite graphs with the vertices of one part indexed by the $(k-1)$-subsets of a $(2k-1)$-set and the vertices of the other part indexed by the k-subsets. Adjacency is given by containment. Thus, B_k is really the middle two levels of the boolean lattice.

Conjecture 13. *The boolean graphs* B_k *are hamiltonian.*

It is also interesting to note that for $k = 2^m$, $m \geq 1$, if O_k has a hamiltonian decomposition, then in fact, so does B_k (D. Duffus, personal communication).

Jaeger [159] proved that if G can be decomposed into an even number of hamiltonian cycles, then its line graph $L(G)$ is 1-factorable. He used the fact that if G

can be decomposed into two hamiltonian cycles, then $L(G)$ can be decomposed into three hamiltonian cycles. This leads to the following conjecture of Bermond [30].

Conjecture 14. *If G has hamiltonian decomposition, then so does* $L(G)$.

Recent results using regularity and line graphs are the following results of Pike:

Theorem 67 [204]. *If G is a* $2k$*-regular graph that has a perfect* 1*-factorization, then the line graph* $L(G)$ *of G is hamiltonian decomposable.*

Theorem 68 [205]. *If G is a bipartite* $(2k+1)$*-regular graph that is hamiltonian decomposable, then* $L(G)$ *is also hamiltonian decomposable.*

5. Toughness

Let $\omega(G)$ denote the number of components of the graph G. A graph G is t-tough if $|S| \geq t\,\omega(G-S)$ for every subset S of the vertex set $V(G)$ with $\omega(G-S) > 1$. The toughness of G, denoted $\tau(G)$, is the maximum t for which G is t-tough (taking $\tau(K_n) = \infty$). Chvátal [78] introduced the idea of toughness. He also raised a problem that has stirred interest ever since.

Problem 7. *Does there exist a* t_0 *such that every* t_0*-tough graph is hamiltonian?*

For a number of years the focus of the investigation was on 2-tough graphs. In [96] it was shown that every k-tough graph of order n with $n \geq k+1$ and kn even has a k-factor. Further, for every $\epsilon > 0$, there exists a $(k-\epsilon)$-tough graph on n vertices with $n \geq k+1$ and kn even which has no k-factor.

However, despite this supporting evidence, Bauer, Broersma, and Veldman[21] were able to show that 2-tough was not enough. Recall a graph is traceable if it contains a spanning path (hamiltonian path).

Theorem 69 [21]. *For every* $\epsilon > 0$, *there exists a* $(9/4-\epsilon)$*-tough nontraceable graph.*

This result calls into question the existence of any such t_0 that will suffice. There does not seem to be a natural candidate for t_0 at the moment.

Despite this development, many other questions remain concerning toughness and hamiltonian properties. One such problem concerns chordal graphs. Recall that a graph is chordal if every cycle of length four or more has a chord. Chvátal[78] produced $(3/2-\epsilon)$-tough chordal graphs without a 2-factor, for arbitrary $\epsilon > 0$. However, recently in [24] it was shown that every 3/2-tough chordal graph has a 2-factor. Thus, the problem remains:

Problem 8. *Is there a* t_0 *such that every* t_0*-tough chordal graph is hamiltonian?*

In [21], it was also shown that there exists a $(7/4 - \epsilon)$-tough chordal nontraceable graph for every $\epsilon > 0$. However, an upper bound was provided by Chen, Jacobson, Kézdy and Lehel in [75], which still leaves a considerable gap for further investigation.

Theorem 70 [75]. *Every* 18*-tough chordal graph is hamiltonian.*

We note that far less toughness is needed for certain special classes of chordal graphs. In [168] it was shown that 1-tough interval graphs are hamiltonian and in [90] it was shown that 1-tough cocomparability graphs are hamiltonian. In [32] it was also established that not all 1-tough chordal planar graphs are hamiltonian, however they also established the following:

Theorem 71 [32]. *Let G be a chordal, planar graph with $\tau(G) > 1$. Then G is hamiltonian.*

The condition that G is chordal is needed above since in [200], nonhamiltonian maximal planar graphs G with $\tau(G) > 3/2 - \epsilon$ for arbitrarily small positive ϵ are constructed.

Recall a graph is *split* if it can be partitioned into an independent set and a clique. In [180] it was shown that every 3/2-tough split graph is hamiltonian and that there is a sequence of split graphs $\{G_k\}_{k=1}^{\infty}$ without 2-factors and with $\tau(G_k) \to 3/2$.

Jung [165] showed that the classic degree condition of Ore could be lowered under a toughness assumption.

Theorem 72. *Let G be a* 1*-tough graph on $n \geq 11$ vertices with $\sigma_2 \geq n - 4$. Then G is hamiltonian.*

In [117], a generalization of the weaker minimum degree condition was obtained (see Theorem 8). Several other generalizations of Jung's Theorem have also been found. Bauer, Chen and Lasser [23] showed that if $n \geq 30$, $\tau(G) > 1$, and $\sigma_2(G) \geq n - 7$, then G is hamiltonian. Fassbender [111] considered degree sums of three independent vertices.

Theorem 73. *If G is a* 1*-tough graph of order $n \geq 13$ such that $\sigma_3 \geq \frac{3n-14}{2}$, then G is hamiltonian.*

Further, the following blend of Ore-type and Fan-type conditions appeared in [22].

Theorem 74. *If G is a* 1*-tough graph of order n such that $\sigma_3(G) \geq n$ and for all $x, y \in V(G)$, $dist(x, y) = 2$ implies that $\max\{\deg(x), \deg(y)\} \geq (n-4)/2$, then G is hamiltonian.*

In [166], the following was shown, generalizing earlier work of Dirac [92] that $c(G) \geq \min\{2\delta, n\}$ and of Bauer, et al [19] that if $n < (t+1)\delta + t + 1$, then G is hamiltonian.

Theorem 75 [166]. *If G is a* 2*-connected t-tough graph with minimum degree* δ, *then* $c(G) \geq \min\{(t+1)\delta + t, n\}$.

Also along these lines, Li [188] and Hoa [149] and [150] also provided bounds on the circumference of 1-tough graphs. Wei [240] also showed the following:

Theorem 76. *Let G be a graph and let*

$$\bar{\sigma}_3 = \min\{\sum_{i=1}^{3} d(u_i) - |\cap_{i=1}^{3} N(u_i)| : u_i, i = 1,2,3 \text{ is an independent set}\}.$$

Then if $\sigma_3(G) \geq n$ *and* $\bar{\sigma}_3 \geq n-4$, *then G is hamiltonian.*

Hoa [148] used toughness and neighborhood conditions together to obtain the following. Here $NC_t(G) = \max\{\cup_{i=1}^{t} N(v_i) \mid v_1, \ldots, v_t \text{ is an independent set}\}$.

Theorem 77. *Every* 1*-tough graph G of order n with* $\sigma_3(G) \geq n$ *contains a cycle of length at least* $\min\{n, 2NC_{\sigma_3 - n+5}\}$.

This implies results of Fassbender [111] and Flandrin, Jung and Li [126] as well as others.

For a broader survey of toughness results see [18].

6. Random Graphs

In this section we present results concerning hamiltonian properties in random graphs. For readers unfamiliar with this subject, I suggest [163] and the survey on matchings and cycles in random graphs by Frieze [131] as starting points. I assume a fundamental knowledge of the area and the models $G_{n,p}$ (binomial random graph on n vertices) and $G_{n,M}$ (uniform random graph on n vertices). Other models will be described as needed.

One general question that received considerable attention over the past decade deals with hamiltonian properties of random regular graphs and digraphs. The first major result here came in a series of two papers by Robinson and Wormald [208] and [209], the first handling the cubic case, the second the general case.

Theorem 78. *For every* $r \geq 3$, *almost all r-regular graphs are hamiltonian.*

Cooper, Frieze and Molloy [86] followed with the digraph case.

Theorem 79. *For every fixed* $r \geq 3$, *almost every r-regular digraph is hamiltonian.*

Another interesting extension is finding hamiltonian cycles containing j specified edges (such as matchings). Recently Robinson and Wormald [210] strengthened their results on r-regular graphs by showing that a random r-regular graph with $j = o(\sqrt{n})$ distinguished edges which are also provided with an orientation, asymptotically almost surely has a hamiltonian cycle containing these edges and respecting the orientations. Further, they showed that a random cubic graph has a hamiltonian cycle that contains the given edges and respects the given orientations of those edges with probability $(e^{-2j^2}/3n) + o(1)$. They also obtain analogs for values of $r \geq 3$.

Recently, Kim and Wormald [169] also considered the following question.

Question 6. *Given a set of randomly generated perfect matchings of an even number of vertices, what is the probability that each of a prescribed set of pairs of those matchings induces a hamiltonian cycle?*

They select four perfect matchings of $2n$ vertices, independently at random. Then they find the asymptotic probability that each of the first and second matchings forms a hamiltonian cycle with each of the third and fourth matchings. They generalize this to any fixed number of matchings, where a prescribed set of matchings must produce hamiltonian cycles. They use this to show that a random r regular graph, for fixed even $r \geq 4$, asymptotically almost surely decomposes into $r/2$ hamiltonian cycles.

In a related vein, let π be a permutation of the set $[n]$. The undirected graph G_π has vertex set $[n]$ and edge set E_π consisting of edges $\{i, j\}$ such that $\pi(i) = j$ or $\pi(j) = i$. Then $\cup_{i=1}^{k} E_{\pi_i}$ is a $2k$-regular multigraph. Frieze [132] showed the following.

Theorem 80. *If π_1 and π_2 are chosen independently and uniformly at random, then $E_{\pi_1} \cup E_{\pi_2}$ is hamiltonian with probability tending to 1 as n tends to infinity.*

For the directed version of this problem, Frieze [132] shows that three permutations suffice and Cooper [83] shows that two permutations do not suffice.

A slightly weaker, but still interesting question is the following:

Question 7. *Given a random graph or digraph, under what conditions can we find multiple edge disjoint hamiltonian cycles?*

In order to consider the question of edge disjoint hamiltonian cycles in random graphs, we define the following. Let $G_{n,m,k}$ denote the class of graphs with n vertices, m edges and minimum degree at least k, with each graph being equiprobable. Also, we say G has the property A_k if G contains $\lfloor (k-1)/2 \rfloor$ edge disjoint hamiltonian cycles, and if k is even, a perfect matching. In [34], Bollobás, Cooper, Fenner and Frieze show the following.

Theorem 81. *Let $k \geq 3$. There exists a constant $C_k \leq 2(k+1)^3$ such that if $2m = cn$, $c \geq C_k$, then with probability tending to 1 as n tends to infinity $G \in G_{n,m,k}$ has property A_k.*

Earlier, Bollobás, Fenner and Frieze [36] established the threshold for the stronger property A_k^*. A graph G has property A_k^* if G contains $\lfloor k/2 \rfloor$ edge disjoint hamiltonian cycles and if k is odd, an edge disjoint perfect matching. In [36] it is shown that for $2m = n(\log n/(k+1) + k\log \log n + d_n)$, the probability $G \in A_k^*$ approaches $0, e^{-\theta_k(d)}$, or 1 respectively for d_n approaching $-\infty$

sufficiently slowly, constant d, or $+\infty$ respectively. They also give an explicit formula for $\theta_k(d)$.

Also using the probabilistic method, Adler, Alon and Ross [3] showed that the maximum number of directed hamiltonian paths in a complete directed graph with n vertices is at least $(e - o(1))(n!/2^{n-1})$.

Now let $H(G)$ denote the number of hamiltonian cycles in G and let $G_{n,r}$ denote the random r-regular graph on n vertices and E the expectation.

Frieze, Jerrum, Molloy, Robinson and Wormald [133] considered r-regular random graphs and showed that with high probability

$$H(G) \geq \frac{1}{n}\sqrt{\frac{\pi}{2n}}\left[(r-1)\frac{r-2^{\frac{r-2}{r}}}{r}\right]^n.$$

Janson [160] found the expected number of hamiltonian cycles for the random graph $G_{n,p}$ and that the hamiltonian cycles have a log-normal distribution. This varies from his findings for tournaments [162] where he showed the following.

Theorem 82. *Let $H(T_n)$ be the number of directed hamiltonian cycles in the random tournament T_n. Then*

$$E(H(T_n)) = (n-1)!2^{-n}$$

and $H(T_n)$ is asymptotically normally distributed.

Using a more general theorem of Janson [161] we obtain the following result.

Theorem 83. *Let $r \geq 3$ be fixed. Then,*

$$\frac{H(G_{n,r})}{E(H(G_{n,r}))} \xrightarrow{d} W = \prod_{\substack{i\geq 3 \\ i \text{ odd}}} (1 - 2/(r-1)^i)^{Z_i} e^{1/i}$$

where $Z_i \in Po((r-1)^i/2i)$ are independent Poisson random variables, and $\xrightarrow{d}$ denotes convergence in distribution.

Note that Theorem 83 may be extended to multigraphs as well. The interested reader should see [163] for a more detailed discussion of this work.

Cooper [81], [82] considered another interesting variation. A given hamiltonian cycle H in a graph is called k-pancyclic if for each s, $(3 \leq s \leq n-1)$ we can find a cycle C of length s using only the edges of H and at most k other edges. Cooper [82] showed that the threshold $p = (\log n + \log\,\log n + c)/n$ (the original threshold for $G_{n,p}$ provided in [177]) for being hamiltonian is also the threshold for the existence of a 1-pancyclic hamiltonian cycle, while in [81] he showed this threshold is also the threshold for a 2-pancyclic hamiltonian cycle.

Broder, Frieze and Shamir [66] considered a random graph G composed of a hamiltonian cycle on n labeled vertices and dn random edges that "hide" the cycle.

They ask the question: Is it possible to efficiently find a hamiltonian cycle in G? Their solution is an $O(n^3 \log n)$-step algorithm and they show that this algorithm succeeds almost surely.

Cooper and Frieze [84] continued the investigations of thresholds by considering the following model. The random digraph $D_{k-in,l-out}$ has vertices $1, 2, \ldots, n$ and each vertex v chooses independently and uniformly at random k arcs into v and l arcs out of v. They show that with probability tending to 1 as $n \to \infty$, the random digraph $D_{3-in,3-out}$ is hamiltonian. While in [85] they show that with probability tending to 1 as n tends to infinity, $D_{2-in,2-out}$ is hamiltonian while $D_{1-in,2-out}$ and $D_{2-in,1-out}$ are not hamiltonian. In particular, this implies that G_{4-out} the underlying graph of $D_{2-in,2-out}$ is hamiltonian, continuing a long line of results of this type. In particular, this improved upon the result of Frieze and Łuczak [135] that G_{5-out} is hamiltonian. Still open is the question of G_{3-out}.

Palmer [202] considered another classic approach to the hamiltonian question, namely the result of Chvátal and Erdös [80] that if the connectivity κ is at least as large as the independence number α, then G is hamiltonian. Palmer showed the following.

Theorem 84. *If G is a random graph with edge probability p given by $p^2 n = c \log n$, then for constant $c > 1$, almost all graphs have $\kappa > \alpha$, and hence are hamiltonian.*

As a consequence, an algorithm of Chvátal [79] for finding a hamiltonian cycle almost always succeeds if $c > 1$.

Finally, Frieze, Karoński and Thoma [134] showed that the probability that the sum of two random trees with $2n$ vertices contains a perfect matching and the sum of five random trees of order n contains a hamiltonian cycle both tend to 1 as n tends to infinity.

7. Forbidden Subgraphs

During the 1980's a number of fundamental results were proved showing that, in a 2-connected graph, when particular pairs of induced subgraphs were forbidden, the graph was hamiltonian. Notable among these were the following results (see Figure 2 for graphs and note that Z_2 is obtained from Z_3 by removing the vertex of degree one).

Theorem 85 [91]. *If G is a $\{K_{1,3}, N\}$-free graph, then*
(a) *if G is 2-connected, then G is hamiltonian*;
(b) *if G is connected, then G is traceable.*

Other notable results similar to Theorem 85 are:

Theorem 86 [65]. *If G is a 2-connected $\{K_{1,3}, P_6\}$-free graph, then G is hamiltonian.*

Theorem 87 [141]. *If G is a 2-connected $\{K_{1,3}, Z_2\}$-free graph, then G is hamiltonian.*

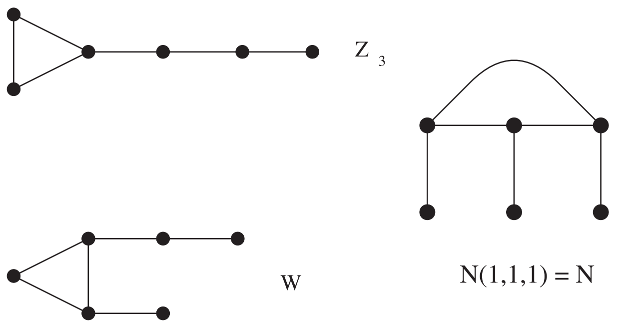

Fig. 2. Common forbidden graphs

Theorem 88 [26]. *If G is a* 2*-connected* $\{K_{1,3}, W\}$*-free graph, then G is hamiltonian.*

I should also point out that recently, in [51], a linear time algorithm for finding a hamiltonian cycle in a $\{K_{1,3}, N\}$-free graph was given. This prompts the following question.

Question 8. *Does a linear time algorithm for finding a hamiltonian cycle exist for any of the other major families described by forbidden pairs, that is, in* $\{K_{1,3}, P_6\}$*-free,* $\{K_{1,3}, Z_3\}$*-free,* $\{K_{1,3}, W\}$*-free or even* $\{K_{1,3}, Z_2\}$*-free graphs?*

Since the completion of the above theorems, this area has experienced a very significant development in both theory and techniques. Far more results have appeared than I will be able to discuss here. Hence, I will limit my presentation mainly to characterizations, strong technique developments and significant open problems and supporting results.

We begin with the work of Bedrossian [26] who characterized all pairs of forbidden graphs which imply all 2-connected such graphs are hamiltonian. In his proof Bedrossian used a nonhamiltonian graph of order 9 to eliminate some possibilities. The four major graphs in his characterization were N, P_6, Z_2 and W, as all others were induced subgraphs of one of these. Later, the following was shown.

Theorem 89 [121]. *If G is a* 2*-connected* $\{K_{1,3}, Z_3\}$*-free graph of order* $n \geq 10$, *then G is hamiltonian.*

This result indicated that if one considers all sufficiently large graphs, something more can be said about the pairs. In [112] this was investigated (for graphs of order $n \geq 10$). We now summarize the combined results of [26] and [112]. We include the proof as an indication of proofs of this type.

Theorem 90 ([26] and [112]). *Let R and S be connected graphs* $(R, S \neq P_3)$ *and G a* 2*-connected graph of order n. Then G is* $\{R, S\}$*-free implies G is hamiltonian if, and only if,* $R = K_{1,3}$ *and S is one of the graphs N,* P_6*, W,* Z_2*, or* Z_3 (*when* $n \geq 10$), *or a connected induced subgraph of one of these graphs.*

Proof. That each of the pairs implies G is hamiltonian follows from Theorems 85, 86, 89 and 88 and our remarks about induced subgraphs of forbidden graphs.

Now consider the graphs $G_0, \ldots, G_6$ of Figure 3. Each is 2-connected and nonhamiltonian. Without loss of generality assume that R is a subgraph of G_1.

Case 1. Suppose that R contains an induced P_4.

Since G_4, G_5, and G_6 are all P_4-free, then S must be an induced subgraph of each of them. But if S is an induced subgraph of G_4, then either S is a star or S contains an induced C_4. However, G_5 is C_4-free, hence S must be a star. Since the only induced star in G_6 is $K_{1,3}$, we have that $S = K_{1,3}$.

Case 2. Suppose that R does not contain an induced P_4.

Then, using G_0 we see immediately that R must be a tree containing at most one vertex of degree 3 and since R contains no induced P_4, we see that $R = K_{1,3}$. Thus, for the remainder of the proof we assume without loss of generality that $R = K_{1,3}$.

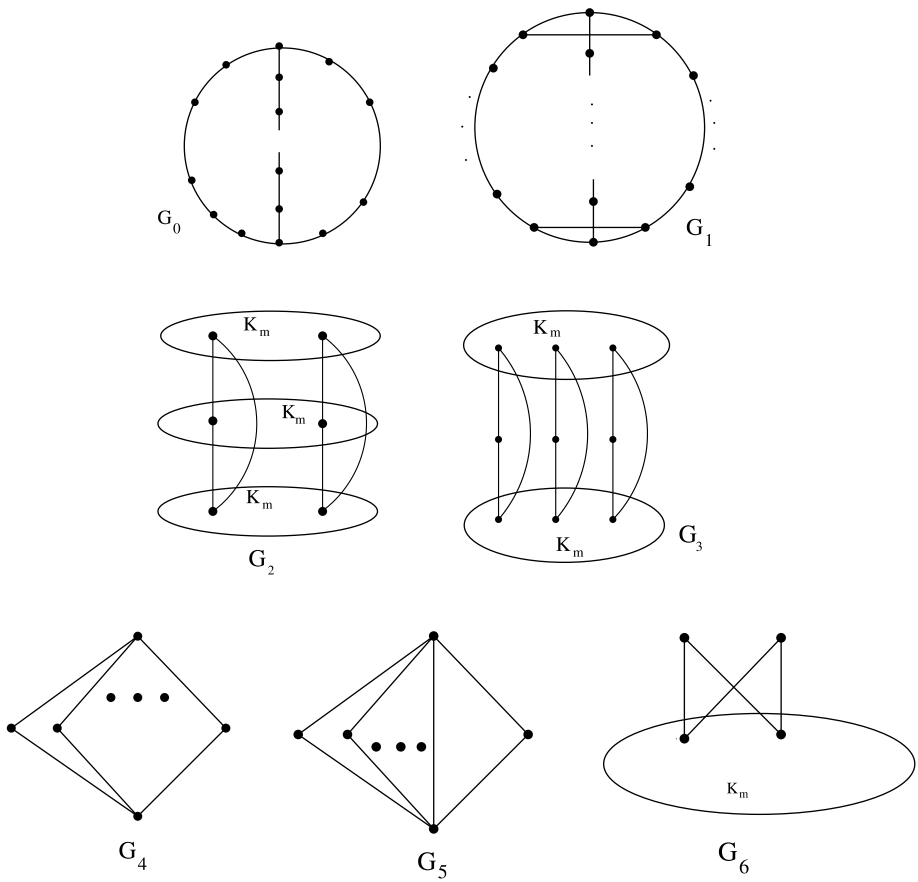

Fig. 3. 2-Connected nonhamiltonian graphs

Now, S must be an induced subgraph of G_1, G_2, and G_3 (each of which is claw-free). The fact that S is an induced subgraph of G_1 implies that S is a path or S is K_3, possibly with a path off each of its vertices. Suppose that S is a path. Since S is an induced subgraph of G_3 which is P_7-free, we see that if S is a path, it is one of P_4, P_5 or P_6.

Hence, we now assume that S contains a K_3, possibly with a path off each of its vertices. Note that G_3 is Z_4-free. Further, any triangle in G_2 with a path of length 3 off one of its vertices can have no paths off its other vertices (leaving Z_3, Z_2, Z_1, and K_3). Again examining G_2 we see it contains no triangle with a path of length 2 from one of its vertices and a path of length 1 from the other two vertices (leaving B or W). Now the graph G_3^* obtained by deleting the edges from the upper K_m to the lower K_m of G_3 is claw-free and contains no induced K_3 with a 2-path off two vertices. The only remaining possibility is a path of length 1 off each of the vertices of K_3, that is, the graph N. □

You may ask why pairs were considered instead of a single graph, but it is an easy observation that P_3 is the only nontrivial single graph that when forbidden implies G is hamiltonian (see [112]). Faudree and Gould [112] went on to characterize the forbidden pairs for several other strong hamiltonian properties.

Theorem 91 [112]. *Let R, S be connected graphs $(R, S \neq P_3)$ and let G $(G \neq C_n)$ be a 2-connected graph of order $n \geq 10$. Then G is $\{R, S\}$-free implies G is pancyclic if, and only if, $R = K_{1,3}$ and S is one of P_4, P_5, P_6, Z_1* or *Z_2.*

Theorem 92 [112]. *Let R, S be connected graphs $(R, S \neq P_3)$ and let G be a 3-connected graph. Then G is $\{R, S\}$-free implies G is panconnected if, and only if, $R = K_{1,3}$ and $S = Z_1$.*

Recently, all pairs that imply all 3-connected graphs are pancyclic were given in [139]. Here, $N_{i,j,k}$ is a graph which consists of K_3 and vertex disjoint paths of length i, j, k rooted at its vertices and L, the graph which consists of two vertex-disjoint copies of K_3 and an edge joining them.

Theorem 93. *Let X and Y be connected graphs on at least three vertices such that $X, Y \neq P_3$ and $Y \neq K_{1,3}$. Then the following statements are equivalent:*

(a) *Every 3-connected $\{X, Y\}$-free graph G is pancyclic.*

(b) *$X = K_{1,3}$ and Y is a subgraph of one of the graphs from the family $\mathcal{F} = \{P_7, L, N_{4,0,0}, N_{3,1,0}, N_{2,2,0}, N_{2,1,1}\}$.*

In each of the above pair results, the claw $K_{1,3}$ must be one of the two graphs. This led naturally to the question: If we consider triples of forbidden subgraphs implying hamiltonicity, must the claw always be one of the three graphs? This was answered negatively in [116] where all triples containing no $K_{1,t}$ with $t \geq 3$ for sufficiently large 2-connected graphs were given. Further, in [113] other forbidden triples for sufficiently large graphs were investigated. Brousek [67] gave the collection of all triples which include the claw that imply all 2-connected graphs are hamiltonian. While in [113], all remaining triples for all graphs were given, thus completing the characterization in this case.

A graph G is said to be *cycle extendable* if any nonhamiltonian cycle can be extended to a cycle containing exactly one more vertex, that is, C is extended to a cycle C' with $V(C') = V(C) \cup \{x\}$ for some vertex x not on C. We say G is *fully cycle extendable* if G is cycle extendable and every vertex of G lies on a triangle. This concept was introduced by Hendry [146]. In that paper he also showed the following:

Theorem 94 [146]. *If G is a* 2*-connected graph of order* $n \geq 10$ *that is* $\{K_{1,3}, Z_2\}$*-free, then G is cycle extendable.*

The cycle extendable pairs were also characterized.

Theorem 95 [112]. *Let R, S be connected graphs* $(R, S \neq P_3)$ *and G a* 2*-connected graph of order* $n \geq 10$*. Then G is* $\{R, S\}$*-free implies G is cycle extendable if, and only if,* $R = K_{1,3}$ *and S is one of* C_3, P_4, Z_1 *or* Z_2*.*

More information on forbidden subgraphs and cycle extendability can be found in [124].

The property of being hamiltonian connected has proven to be more elusive. Shepherd [224] considered 3-connected claw-free graphs.

Theorem 96 [224]. *If G is a* 3*-connected* $\{K_{1,3}, N\}$*-free graph, then G is hamiltonian-connected.*

However, no complete characterization of pairs for hamiltonian-connected graphs is known. The next result is from [112].

Theorem 97. *Let R, S be connected graphs* $(R, S \neq P_3)$ *and let G be a* 3*-connected graph. If G is* $\{R, S\}$*-free implies G is hamiltonian-connected, then* $R = K_{1,3}$ *and S satisfies each of the following:*
(a) $\Delta(S) \leq 3$.
(b) *The longest induced path in S is at most a* P_{12}.
(c) *S contains no cycles except for* C_3.
(d) *All triangles in S are vertex disjoint.*
(e) *S is claw-free.*

(*Note.* there are only a finite number of possible graphs for S).

Besides the progress made in characterizing pairs and triples for various hamiltonian properties, a new and powerful tool for dealing with hamiltonian problems in claw-free graphs was developed by Ryjáček [212]. This tool has not only allowed people to attack new questions, but also provided ways to prove a number of the previously established results in much simpler ways.

For a vertex x such that $G[N(x)]$ is connected, a local completion of G at x means the graph obtained by replacing $G[N(x)]$ by a clique on $N(x)$. Ryjáček [212] then showed the local completion was well defined and if G was claw-free then the resulting graph was claw-free. With this result, a closure operation was now possible. This graph is called the closure of G and is denoted $cl(G)$. (Note this graph is different from the well-known degree sum closure due to Bondy and Chvátal [45] or any of several other closures that have been developed. For more information on closures, see [63], [62] and [214].) Ryjáček's main result is:

Theorem 98 [212]. *Let G be a claw-free graph. Then*
(a) *the closure* $cl(G)$ *is well-defined,*
(b) *there is a triangle-free graph* H *such that* $cl(G) = H$,
(c) $c(G) = c(cl(G))$, *where* $c(G)$ *is the circumference of* G.

Now, for a class C we say that C is stable under closure (or simply stable) if $cl(G) \in$ C for every $G \in$ C. Ryjáček [212] then had proved the following important result.

Theorem 99. *The length of a longest cycle and hamiltonicity are stable properties in the class of claw-free graphs.*

The question remained as to the stability of other hamiltonian type properties. Several of these were studied in [57] where it was shown that in the class of k-connected claw-free graphs, pancyclicity, vertex pancyclicity and cycle extendability are not stable for any k. Further, traceability is stable (even for $k = 1$) and homogeneous traceability is not stable for $k = 2$ although it is stable for $k = 7$.

Several interesting conjectures thus remain. For example: The property of being hamiltonian-connected is stable in the class of claw-free graphs. This conjecture was recently proved by Brandt [54] for $k = 9$. However, the question remains as to the minimum possible k.

Theorem 100 [54]. *Every* 9*-connected claw-free graph is hamiltonian connected.*

Also, still remaining is the following problem.

Problem 9. *Determine the smallest integer k for which the property of being homogeneously traceable is stable in the class of k-connected claw-free graphs.*

By modifying the closure idea, it was shown in [37] that stability can be obtained. Their idea was to do a local completion only when a vertex was locally k-connected rather than just locally connected, that is, when $G[N(x)]$ is k-connected. With this modified closure, denoted $cl_k(G)$, they were able to show the following:

Theorem 101. *Let G be a claw-free graph. Then*
(a) $cl_k(G)$ *is uniquely determined;*
(b) G *is hamiltonian connected if and only if* $cl_3(G)$ *is hamiltonian connected, and*
(c) G *is homogeneously traceable if and only if* $cl_2(G)$ *is homogeneously traceable.*

Further, they made the following conjecture.

Conjecture 15. *Let G be a claw-free graph. Then G is hamiltonian connected if and only if $cl_2(G)$ is hamiltonian connected.*

In [213] it was shown that every claw-free graph with a complete closure contains an $(n-1)$-cycle. The following was also conjectured:

Conjecture 16. *Let G be a claw-free graph of order n whose closure is complete and let c_1 and c_2 be fixed constants. Then for sufficiently large n, the graph G contains cycles C_i for all i with $3 \leq i \leq c_1$ and $n - c_2 \leq i \leq n$.*

In [128], nonhamiltonian closed claw-free graphs with small clique covering number were studied. Ryjáček [212] also applied his closure to show the following:

Theorem 102. *Every* 7*-connected claw-free graph is hamiltonian.*

This still leaves open the fundamental conjecture of Matthews and Sumner [192].

Conjecture 17. *Every* 4*-connected claw-free graph is hamiltonian.*

In [60] it was shown that this conjecture holds in the class of hourglass-free (2 triangles sharing a single vertex) graphs. Further, it was shown that a weaker form of the conjecture holds, namely one with hamiltonian replaced by a connected spanning subgraph in which each vertex has degree two or four.

The Matthews-Sumner conjecture is equivalent to an earlier conjecture due to Thomassen [232].

Conjecture 18. *Every* 4*-connected line graph is hamiltonian.*

Besides Brandt's result (Theorem 100), the following results have been shown.

Theorem 103 [245]. *Every line graph of a* 4*-edge connected graph is hamiltonian.*

Theorem 104 [184]. *Every* 4*-connected line graph of a planar graph is hamiltonian.*

Recently, Kriesell [182] showed the next result.

Theorem 105. (a) *Every* 4*-connected line graph of a claw-free graph is hamiltonian connected.*
(b) *Every* 4*-connected hourglass-free line graph is hamiltonian connected.*

Conjecture 18 is still unsolved even when restricted to classes such as 5 or 6-regular graphs. Saito (see [183]) considered graphs of small diameter and made the next conjecture.

Conjecture 19. *Every* 3*-connected line graph of diameter at most* 3 *is hamiltonian unless it is the line graph of a graph obtained from the Petersen graph by adding at least one pendant edge to each of its vertices.*

As evidence of this conjecture, Kriesell [183] showed that every 3-connected line graph of diameter at most 3 has a hamiltonian path.

8. Special Topics

In this section we consider a few special problems that do not fit directly into the other sections. We begin with the following idea. Let G^c denote a graph G whose edges are colored in an arbitrary way. A properly colored cycle in G^c is a cycle in which adjacent edges have different colors. Such cycles are termed *alternating cycles*. In particular, we are interested in alternating hamiltonian cycles.

Bollobás and Erdős [35] considered this problem in colored complete graphs K_n^c, that is, a complete graph colored with c colors. Let $\Delta(G^c)$ denote the maximum degree in G^c in any one color. Bollobás and Erdős [35] made the following conjecture.

Conjecture 20. *Every K_n^c with $\Delta(K_n^c) \leq \lfloor n/2 \rfloor - 1$ contains an alternating hamiltonian cycle.*

This conjecture, if true, would provide a sharp bound, as can be seen by letting $n = 4k + 1$. Then there clearly exists a K_n^2 so that both its monochromatic subgraphs are regular of degree $2k$. However, such a graph is clearly not alternating hamiltonian, as n is odd.

Bollobás and Erdős [35] were able to show that if $\Delta(K_n^c) < n/69$, then it contains an alternating hamiltonian cycle. This was later improved by Chen and Daykin [69] who showed $\Delta(K_n^c) \leq n/17$ worked and then by Shearer [222] who showed $\Delta(K_n^c) < n/7$. The best known result is the recent improvement due to Alon and Gutin [7].

Theorem 106. *For every $\epsilon > 0$ there exists an $n_0 = n_0(\epsilon)$ so that, for every $n > n_0$, every K_n^c satisfying*

$$\Delta(K_n^c) \leq (1 - 1/\sqrt{2} - \epsilon)n$$

contains an alternating hamiltonian cycle.

Another somewhat unexpected result is due to Barr [17].

Theorem 107. *Every K_n^c without monochromatic triangles contains an alternating hamiltonian path.*

The 2-color version of this general question has been considered separately. Manoussakis (see [215]) posed the problem of finding a polynomial algorithm for finding a longest alternating cycle in a 2-edge colored complete graph. This was recently answered affirmatively in [15]. Earlier, Saad [215] proved the existence of a randomized polynomial algorithm for the problem. A natural problem now presents itself and was given in [27].

Problem 10. *Determine the complexity of the alternating hamiltonian cycle problem for c-edge colored complete graphs when $c \geq 3$.*

Clearly we can ask a similar question in other important classes of graphs. A related problem is due to Bang-Jensen and Gutin [14].

Problem 11. *Determine the maximum bounds t_1 and t_2 such that $K^c_{m,m}$ satisfying $\Delta(K^c_{m,m}) \leq t_1$ and $\Delta(K^c_{m,m}) \leq t_2$ is alternating hamiltonian and even-pancyclic, respectively.*

For a more complete treatment of this general area see [14].

Next, suppose G is a *weighted graph*, that is, each edge e of G is assigned a nonnegative number $w(e)$, called the *weight* of e. Let the weighted degree $\deg^w x = \sum w(e)$, where the sum is taken over all edges incident to x. Bondy and Fan [46] gave a Dirac-type result for weighted graphs.

Theorem 108. *Let G be a 2-connected weighted graph such that $\deg^w x \geq r$ for every vertex x of G. Then, either G contains a cycle of weight at least 2r, or every cycle of maximum weight in G is a hamiltonian cycle.*

Note that this result is no longer valid if we permit negative weights. To see this, subdivide each edge of $K_{2,3}$ $m \geq 1$ times and weight each edge of the resulting graph with -1.

Bondy and Fan [46] also raised several natural questions about extending their result. These questions were all answered negatively. However, with some modification of the type of conclusion desired, further results are possible. For example, Bondy, Broersma, van den Heuvel and Veldman [44] were able to show the following.

Theorem 109. *Let G be a 2-connected weighted graph such that $\sigma^w_2(G) \geq s$. Then G contains either a cycle of weight at least s or a hamiltonian cycle.*

Bondy and Fan also conjectured that weighted versions of the results of Erdős and Gallai [103], who proved that every graph of order n contains a path of length at least $2q/n$ and, provided $q \geq n$, a cycle of length $2q/(n-1)$ existed.

Frieze, McDiarmid and Reed [136] proved that every weighted graph contains a "heavy path".

Theorem 110. *Let G be a weighted graph of order n. Then G contains a path of weight at least $2w(G)/n$.*

Bondy and Fan [47] provided the following theorem on heavy cycles.

Theorem 111. *Let G be a weighted 2-edge connected graph of order n. Then G contains a cycle of weight at least $2w(G)/(n-1)$.*

Bollobás and Scott [38] provided extensions of the theorems of both Dirac and also Erdős and Gallai to digraphs. Finally, a weighted extension of a result of Enomoto was found in [247].

9. Suppose *G* is Hamiltonian

In 1982, Mitchem and Schmeichel [194] suggested that the degree bounds in theorems guaranteeing pancyclicity or bipancyclicity (that is, a bipartite graph containing all even cycles from 4 to the order of the graph) could be lowered if hamiltonicity were assumed. This is clearly a strengthening over simply assuming G is 2-connected. As it turns out, Faudree, Häggkvist and Schelp [122] had already asked a question of this type.

Theorem 112 [122]. *If G is a hamiltonian graph on n vertices with $q > \lfloor (n-1)^2/4 \rfloor + 1$ edges, then G is either pancyclic or bipartite.*

Then, in 1981, Amar, Flandrin Fournier, and Germa [11] showed the following:

Theorem 113. *Let G be a hamiltonian, nonbipartite graph of order $n \geq 162$. If $\delta(G) \geq (2n+1)/5$, then G is pancyclic.*

Hakimi and Schmeichel [220] showed that the edge density could be reduced even more by considering a consecutive pair of vertices.

Theorem 114. *If G is a hamiltonian graph of order $n \geq 3$ and if x and y are adjacent vertices on a hamiltonian cycle in G such that $\deg x + \deg y \geq n$, then G is pancyclic, bipartite, or missing only an $(n-1)$-cycle.*

While Entringer and Schmeichel [99] gave a purely bipartite version of the Faudree, Häggkvist and Schelp [122] result.

Theorem 115 [99]. *Let G be a hamiltonian bipartite graph on $2n$ vertices and $q > n^2/2$ edges. Then G is bipancyclic.*

This result was followed by several others of this type. Shi [225] showed that if G is a graph of order $n \geq 40$ and $\deg x + \deg y > 4n/5$, then either G is pancyclic or bipartite. This result is also best possible as can be seen by taking five k-sets of independent vertices and cyclically joining all vertices in one set to all in the next set. This graph has degree sum $4n/5$ but lacks triangles.

Zhang [246] showed the following variation in which it suffices to consider one vertex and all its nonneighbors. He also considered a bipartite version.

Theorem 116 [246]. *If G is a hamiltonian graph of order n and there exists a vertex x such that $\deg x + \deg y \geq n$ for each y not adjacent to x, then either G is pancyclic or $K_{n/2,n/2}$.*

Theorem 117 [246]. *If $G = (X \cup Y, E)$ is a hamiltonian bipartite graph with $|X| = |Y| = n > 3$ and there exists a vertex $x \in X$ such that $\deg x + \deg y \geq n + 1$ for each $y \in Y$ not adjacent to x, then G is bipancyclic.*

Tian and Zang [236] considered a bipartite variation using the hamiltonian assumption.

Theorem 118. *If G is a hamiltonian bipartite graph on* $2n$ *vertices where* $n \geq 60$ *and* $\delta(G) > 2n/5 + 2$, *then G is bipancyclic.*

Finally, these ideas have recently been extended to weakly and semipancyclic graphs in [115] and [138]. We close by posing the following more general problem.

Problem 12. *Given a result that assumes G is* 2*-connected and has properties* $P_1, \ldots, P_k$ *to obtain property P, when does the assumption of hamiltonicity instead of* 2*-connectivity allow us to lessen the other assumptions and obtain the same result?*

References

1. Abbasi, S.: Spanning cycles in dense graphs. Ph.D. thesis, Rutgers University 1999
2. Abderrezzak, M.E.K., Flandrin, E., Amar, D.: Cyclability and pancyclability in bipartite graphs. Discrete Math. **236**, 3–11 (2001)
3. Adler, I., Alon, N., Ross, S.M.: On the maximum number of hamiltonian paths in tournaments. Random Struct. Algorithms **18**, 291–296 (2001)
4. Aigner, M., Brandt, S.: Embedding arbitrary graphs of maximum degree two. J. Lond. Math. Soc., Ser. II. **48**, 39–51 (1993)
5. Aldred, R.E.L., Holton, D.A., Min, Z.K.: A degree characterization of pancyclicity. Discrete Math. **127**, 23–29 (1994)
6. Alon, N., Fischer, E.: 2-Factors in dense graphs. Discrete Math. **152**, 13–23 (1996)
7. Alon, N., Gutin, G.: Properly colored hamilton cycles in edge colored complete graphs. Random Struct. Algorithms **11**, 179–186 (1997)
8. Alon, N., Yuster, R.: H-factors in dense graphs. J. Comb. Theory, Ser. B **66**, 269–282 (1996)
9. Alspach, B., Bermond, J.-C., Sotteau, D.: Decompositions into cycles I. Cycles and rays (Montreal, 1987), NATO ASI Ser., Ser. C **301**, 9–18 (1990)
10. Alspach, B., Heinrich, K., Mohar, B.: A note on hamilton cycles in block-intersection graphs. Finite Geom. Comb. Designs – Contemp. Math. **111**, 1–4 (1990)
11. Amar, D., Flandrin, E., Fournier, I., Germa, A.: Hamiltonian pancyclic graphs. Discrete Math. **46**, 327 (1983)
12. Ainouche, A., Schiermeyer, I.: Insertible vertices, neighborhood intersections, and hamiltonicity. J. Graph Theory **20**, 123–135 (1995)
13. Balakrishnan, R., Paulrajam, P.: Hamiltonian cycles in tensor product of graphs. Discrete Math. **186**, 1–13 (1998)
14. Bang-Jensen, J., Gutin, G.: Alternating cycles and paths in edge-colored multigraphs: a survey. Discrete Math. **165–166**, 39–60 (1997)
15. Bang-Jensen, J., Gutin, G.: Alternating cycles and trails in 2-edge colored complete multigraphs. Discrete Math. **188**, 61–72 (1998)
16. Baranyai, Z., Szasz, Gy. R.: Hamiltonian decompositions of lexicographic product. J. Comb. Theory, Ser. B **31**, 253–261 (1981)
17. Barr, O.: Properly coloured hamiltonian paths in edge-colored complete graphs without monochromatic triangles. Ars Comb. **50**, 316–318 (1998)
18. Bauer, D., Broersma, H.J., Schmeichel, E.: More progress on tough graphs – the Y2K report. (preprint)
19. Bauer, D., Broersma, H.J., van den Heuvel, J., Veldman, H.J.: Long cycles in graphs with prescribed toughness and minimum degree. Discrete Math. **141**, 1–10 (1995)

20. Bauer, D., Broersma, H.J., Li, R., Veldman, H.J.: A generalization of a result of Häggkvist and Nicoghossian. J. Comb. Theory, Ser. B **47**, 237–243 (1989)
21. Bauer, D., Broersma, H.J., Veldman, H.J.: Not every 2-tough graph is hamiltonian. Discrete Appl. Math. **99**, 317–321 (2000)
22. Bauer, D., Broersma, H.J., Veldman, H.J.: On generalizing a theorem of Jung. Ars Comb. **40**, 207–218 (1995)
23. Bauer, D., Chen, G., Lasser, L.: A degree condition for hamiltonian cycles in t-tough graphs with $t > 1$. Advances in Graph Theory, pp. 19–32, Gulbarga: Vishwa 1991
24. Bauer, D., Katona, G.Y., Kratsch, D., Veldman, H.J.: Chordality and 2-factors in tough graphs. Discrete Appl. Math. **99**, 323–329 (2000)
25. Bauer, D., Schmeichel, E., Veldman, H.J.: Some recent results on long cycles in tough graphs. Graph Theory, combinatorics, and applications, Vol 1 (Kalamazoo, MI, 1988) pp. 113–123, New York: Wiley 1991
26. Bedrossian, P.: Forbidden subgraph and minimum degree conditions for hamiltonicity. Ph.D. Thesis, Memphis State University 1991
27. Benkouar, A., Manoussakis, Y., Paschos, V., Saad, R.: On the complexity of finding alternating hamiltonian and eulerian cycles in edge-colored graphs. Lect. Notes Comput. Sci. **557**, 190–198 (1991)
28. Bermond, J.-C.: Hamiltonian graphs. In: Beineke and Wilson, Selected Topics in Graph Theory, London: Academic Press 1978
29. Bermond, J.-C.: Hamilton decomposition of graphs, directed graphs and hypergraphs. In: Advances in Graph Theory. Ann. Discrete Math. **3**, 21–28 (1978)
30. Bermond, J.-C.: Problem 97. Discrete Math. **71**, 275–276 (1988)
31. Bermond, J.-C., Thomassen, C.: Cycles in digraphs – a survey. J. Graph Theory **5**, 1–43 (1981)
32. Böhme, T., Harant, J., Tkáč, M.: More than 1-tough chordal planar graphs are hamiltonian. J. Graph Theory **32**, 405–410 (1999)
33. Bollobás, B., Brightwell, G.: Cycles through specified vertices. Combinatorica **13**, 147–155 (1993)
34. Bollobás, B., Cooper, C., Fenner, T.I., Frieze, A.M.: Edge disjoint hamilton cycles in sparse random graphs of minimum degree at least k. J. Graph Theory **34**, 42–59 (2000)
35. Bollobás, B., Erdős, P.: Alternating hamiltonian cycles. Isr. J. Math. **23**, 126–131 (1976)
36. Bollobás, B., Fenner, T.I., Frieze, A.M.: Hamilton cycles in random graphs of minimal degree at least k. A tribute to Paul Erdős, Cambridge: Cambridge University Press 1990
37. Bollobás, B., Riordan, O., Ryjáček, Z., Saito, S., Schelp, R.: Closure and hamiltonian-connectivity of claw-free graphs. Discrete Math. **195**, 67–80 (1999)
38. Bollobás, B., Scott, A.D.: A proof of a conjecture of Bondy concerning paths in weighted digraphs. J. Comb. Theory, Ser. B **66**, 283–292 (1996)
39. Bollobás, B., Thomason, A.: Highly linked graphs, Combinatorica **16**, 313–320 (1996)
40. Bollobás, B., Thomason, A.: Weakly pancyclic graphs. J. Comb. Theory, Ser. B **77**, 121–137 (1999)
41. Bondy, J.A.: Pancyclic graphs. J. Comb. Theory, Ser. B **11**, 80–84 (1977)
42. Bondy, J.A.: Hamilton cycles in graphs and digraphs. (Proc. 9th S.E. Conf. Comb., Graph Theory and Computing) Congr. Numerantium **21**, 3–28 (1978)
43. Bondy, J.A.: Basic graph theory – paths and cycles. Handbook of Combinatorics, Vol. I, pp. 5–110, Amsterdam: Elsevier 1995
44. Bondy, J.A., Broersma, H., van den Heuvel, J., Veldman, H.J.: Heavy cycles in weighted graphs. Discuss. Math.: Graph Theory (to appear)
45. Bondy, J.A., Chvátal, V.: A method in graph theory. Discrete Math. **15**, 111–135 (1976)
46. Bondy, J.A., Fan, G.: Optimal paths and cycles in weighted graphs. Ann. Discrete Math. **41**, 53–70 (1989)
47. Bondy, J.A., Fan, G.: Cycles in weighted graphs. Combinatorica, **11**, 191–205 (1991)

48. Bondy, J.A., Jackson, B.: Vertices of small degree in uniquely hamiltonian graphs. J. Comb. Theory, Ser. B **74**, 265–275 (1998)
49. Bosák, J.: Decompositions of Graphs. Dordrecht: Kluwer Academic Publishers 1990
50. Bosák, J.: Hamiltonian lines in cubic graphs. Theory of Graphs (Int. Symp., Rome, 1966), pp. 35–46, New York: Gordan and Breach 1967
51. Brandstädt, A., Dragan, F.F., Köhler, E.: Linear time algorithms for hamiltonian problems on (claw, net)-free graphs. SIAM J. Comput. **30**, 1662–1677 (2000)
52. Brandt, S.: Cycles and paths in triangle-free graphs. In: R. Graham, J. Nesetril: The Mathematics of Paul Erdős, pp. 32–42, Springer 1996
53. Brandt, S.: A sufficient condition for all short cycles. Discrete Appl. Math. **79**, 63–66 (1997)
54. Brandt, S.: 9-Connected claw-free graphs are hamilton-connected. J. Comb. Theory, Ser. B **75**, 167–173 (1999)
55. Brandt, S., Faudree, R.J., Goddard, W.: Weakly pancyclic graphs. J. Graph Theory **27**, 141–176 (1998)
56. Brandt, S., Chen, G., Faudree, R.J., Gould, R.J., Lesniak, L.: On the number of cycles in a 2-factor. J. Graph Theory **24**, 165–173 (1997)
57. Brandt, S., Favaron, O., Ryjáček, Z.: Closure and stable hamiltonian properties in claw-free graphs. J. Graph Theory **34**, 30–41 (2000)
58. Brandt, S., Veldman, H.J.: Degree sums for edges and cycle lengths in graphs. J. Graph Theory **25**, 253–256 (1997)
59. Broersma, H.: On some intriguing problems in hamiltonian graph theory – a survey. Discrete Math. **251**, 47–69 (2002)
60. Broersma, H., Kriesell, M., Ryjáček, Z.: On factors of 4-connected claw-free graphs. J. Graph Theory **37**, 125–136 (2001)
61. Broersma, H., Li, H., Li, J.P., Tian, F., Veldman, H.J.: Cycles through subsets with large degree sums. Discrete Math. **171**, 43–54 (1997)
62. Broersma, H., Ryjáček, Z.: Strengthening the closure concept in claw-free graphs. Discrete Math. **233**, 55–63 (2001)
63. Broersma, H., Ryjáček, Z., Schiermeyer, I.: Closure concepts: a survey. Graphs Comb. **16**, 17–48 (2000)
64. Broersma, H., van den Heuvel, J., Veldman, H.J.: A generalization of Ore's theorem involving neighborhood unions. Discrete Math. **122**, 37–49 (1993)
65. Broersma, H.J., Veldman, H.J.: Restrictions on induced subraphs ensuring hamiltonicity or pancyclicity of $K_{1,3}$-free graphs. In: R. Bodendiek: Contemporary Methods in Graph Theory, pp. 181–194, Mannheim Wien Zurich: BI-Wiss.-Verl. 1990
66. Broder, A.Z., Frieze, A.M., Shamir, E.: Finding hidden hamiltonian cycles. Random Struct. Algorithms **5**, 395–410 (1994)
67. Brousek, J.: Forbidden triples and hamiltonicity. Discrete Math. **251**, 71–76 (2002)
68. Chartrand, G., Lesniak, L.: Graphs & Digraphs. Third Edn. London: Chapman & Hall 1996
69. Chen, C.C., Daykin, D.E.: Graphs with hamiltonian cycles having adjacent lines different colors. J. Comb. Theory, Ser. B **21**, 135–139 (1976)
70. Chen, G., Faudree, R.J., Gould, R.J., Jacobson, M.S., Lesniak, L., Pfender, F.: Linear forests and ordered cycles. (preprint)
71. Chen, G., Faudree, R.J., Gould, R.J., Lesniak, L., Jacobson, M.S.: Cycles in 2-factors of balanced bipartite graphs. Graphs Comb. **16**, 67–80 (2000)
72. Chen, G., Faudree, J.R., Gould, R.J., Saito, A.: 2-Factors in claw-free graphs. Discuss. Math. Graph Theory **20**, 165–172 (2000)
73. Chen, G., Gould, R.J., Pfender, F.: New conditions for k-ordered hamiltonian graphs. (preprint)
74. Chen, G., Jacobson, M.S.: Degree sum conditions for hamiltonicity on k-partite graphs. Graphs Comb. **13**, 325–343 (1997)
75. Chen, G., Jacobson, M.S., Kézdy, A.E., Lehel, J.: Tough enough chordal graphs are hamiltonian. Networks **31**, 29–38 (1998)

76. Chen, G., Liu, Y.: Hamiltonian graphs with large neighborhood unions. Ars Comb. **46**, 227–238 (1997)
77. Chen, G., Schelp, R.: Hamiltonian graphs with neighborhood intersection. J. Graph Theory **18**, 497–513 (1994)
78. Chvátal, V.: Tough graphs and hamiltonian circuits. Discrete Math. **5**, 215–228 (1973)
79. Chvátal, V.: Hamiltonian cycles. In: E.L. Lawler, et al.: The Traveling Salesman Problem, John Wiley & Sons Ltd. 1985
80. Chvátal, V., Erdős, P.: A note on hamiltonian circuits. Discrete Math. **2**, 111–113 (1972)
81. Cooper, C.: Pancyclic hamilton cycles in random graphs. Discrete Math. **91**, 141–148 (1991)
82. Cooper, C.: 1-Pancyclic hamilton cycles in random graphs. Random Struct. Algorithms **3**, 277–287 (1992)
83. Cooper, C.: The union of two random permutations does not have a directed hamiltonian cycle. Random Struct. Algorithms **18**, 95–98 (2001)
84. Cooper, C., Frieze, A.M.: Hamilton cycles in a class of random directed graphs. J. Comb. Theory, Ser. B **62**, 151–163 (1994)
85. Cooper, C., Frieze, C.M.: Hamilton cycles in random graphs and directed graphs. Random Struct. Algorithms **16**, 369–401 (2000)
86. Cooper, C., Frieze, A.M., Molloy, M.: Hamilton cycles in random regular digraphs. Comb. Probab. Comput. **3**, 39–49 (1994)
87. Corrádi, K., Hajnal, A.: On the maximal number of independent circuits in a graph. Acta Math. Acad. Sci. Hung. **14**, 423–439 (1963)
88. Curran, S.J., Gallian, J.A.: Hamiltonian cycles and paths in cayley graphs and digraphs – a survey. Discrete Math. **156**, 1–18 (1996)
89. Dean, N.: Which graphs are pancyclic modulo *k*. Graph theory, combinatorics, and applications, Vol. 1 (Kalamazoo, MI 1988, pp. 315–325) New York: Wiley 1991
90. Deogun, J.S., Kratsch, D., Steiner, G.: 1-Tough cocomparability graphs are hamiltonian. Discrete Math. **170**, 99–106 (1997)
91. Duffus, D., Gould, R.J., Jacobson, M.S.: Forbidden subgraphs and the hamiltonian theme. In: Chartrand, et al.: The Theory and Applications of Graphs, (pp. 297–316) 1981
92. Dirac, G.A.: Some theorems on abstract graphs. Proc. Lond. Math. Soc. **2**, 69–81 (1952)
93. Egawa, Y.: Edge-disjoint hamiltonian cycles in graphs of Ore type. SUT J. Math. **29**, 15–50 (1993)
94. Ellingham, M.N., Zha, X., Zhang, Y.: Spanning 2-trails from degree sum conditions. (preprint)
95. El-Zahar, M.H.: On circuits in graphs. Discrete Math. **50**, 227–230 (1984)
96. Enomoto, H., Jackson, B., Katerinis, P., Saito, A.: Toughness and the existence of *k*-factors. J. Graph Theory **9**, 87–95 (1985)
97. Enomoto, H., Kaneko, A., Kouider, M., Tuza, Zs.: Degree sums and covering cycles. J. Graph Theory **20**, 419–422 (1995)
98. Enomoto, H., Kaneko, A., Tuza, Zs.: P_3-factors and covering cycles in graphs of minimum degree $n/3$. Colloq. Math. Soc. János Bolyai **52**. Combinatorics, pp. 213–220, Eger (Hungary): North Holland 1988
99. Entringer, R., Schmeichel, E.: Edge conditions and cycle structure in bipartite graphs. Ars Comb. **26**, 229–232 (1988)
100. Entringer, R., Swart, H.: Spanning cycles of nearly cubic graphs. J. Comb. Theory, Ser. B **29**, 303–309 (1980)
101. Erdős, P.: Problem 9. In: M. Fielder: Graph Theory and its applications, pp. 159, Prague: Czech. Acad. Sci. Publ. 1964
102. Erdős, P.: Some recent combinatorial problems. Technical Report, University of Bielefeld 1990

103. Erdős, P., Gallai, T.: On maximal paths and circuits. Acta Math. Sci. Hung. **10**, 337–356 (1959)
104. Erdős, P., Lovász, L.: Problems and results on 3-chromatic hypergraphs and some related questions. Colloq. Math. Soc. János Bolyai **10**, 609–627 (1973)
105. Fan, G.H.: New sufficient condition for cycles in graphs. J. Comb. Theory, Ser. B **37**, 221–227 (1984)
106. Fan, C., Liu, J.: Hamiltonian decompositions of strong products. J. Graph Theory **29**, 45–55 (1998)
107. Fan, G.H., Häggkvist, R.: The square of a hamiltonian cycle. SIAM J. Discrete Math. **7**, 203–212 (1994)
108. Fan, G.H., Kierstead, H.: The square of paths and cycles. J. Comb. Theory, Ser. B **63**, 55–64 (1995)
109. Fan, G.H., Kierstead, H.: Partitioning a graph into two square – cycles. J. Graph Theory **23**, 241–256 (1996)
110. Fan, G.H., Kierstead, H.: Hamiltonian square paths. J. Comb. Theory, Ser. B **67**, 167–182 (1996)
111. Fassbender, B.: A sufficient condition on degree sums of independent triples for hamiltonian cycles in 1-tough graphs. Ars Comb. **33**, 300–304 (1992)
112. Faudree, R.J., Gould, R.J.: Characterizing forbidden pairs for hamiltonian properties. Discrete Math. **173**, 45–60 (1997)
113. Faudree, R.J., Gould, R., Jacobson, M.S.: Forbidden triples implying hamiltonicity: for all graphs. (preprint)
114. Faudree, J.R., Faudree, R.J., Gould, R.J., Jacobson, M.S., Lesniak, L.: On *k*-ordered graphs. J. Graph Theory **35**, 69–82 (2000)
115. Faudree, J.R., Faudree, R.J., Gould, R.J., Jacobson, M.S., Lesniak, L.: Variations of pancyclic graphs. (preprint)
116. Faudree, R.J., Gould, R.J., Jacobson, M.S., Lesniak, L.: Characterizing forbidden clawless triples implying hamiltonian graphs. Discrete Math. **249**, 71–81 (2002)
117. Faudree, R.J., Gould, R.J., Jacobson, M.S., Lesniak, L., Saito, A.: Toughness, degrees and 2-factors. (preprint)
118. Faudree, R.J., Gould, R.J., Jacobson, M.S., Schelp, R.: Seymour's Conjecture. In: V.R. Kulli: Advances in Graph Theory, pp. 163–171, Gulbarga: Vishwa 1991
119. Faudree, R.J., Gould, R.J., Kostochka, A., Lesniak, L., Schiermeyer, I., Saito, A.: Degree conditions for *k*-ordered hamiltonian graphs. (preprint)
120. Faudree, R.J., Gould, R.J., Lesniak, L., Lindquester, T.: Generalized degree conditions for graphs with bounded independence number. J. Graph Theory **19**, 397–409 (1995)
121. Faudree, R.J., Gould, R.J, Ryjáček, Z., Schiermeyer, I.: Forbidden subgraphs and pancyclicity. Proc. Twenty-sixth Southeastern Conf. Comb. Graph Theory and Computing (Boca Raton, FL 1995) Congr. Numerantium **109**, 13–32 (1995)
122. Faudree, R.J., Häggkvist, R., Schelp, R.: Pancyclic graphs – connected ramsey numbers. Ars Comb. **11**, 37–49 (1981)
123. Faudree, R.J., Rousseau, C., Schelp, R.: Edge disjoint hamiltonian cycles. Graph theory with Applications to Algorithms and Computer Science (Kalamazoo, MI. pp. 231–249, 1984), New York: Wiley 1985
124. Faudree, R.J., Ryjáček, Z., Schiermeyer, I.: Forbidden subgraphs and cycle extendability. J. Comb. Math. Comb. Comput. (to appear)
125. Faudree, R.J., van den Heuvel, J.: Degree sums, *k*-factors and hamilton cycles in graphs. Graphs Comb. **11**, 21–28 (1995)
126. Flandrin, E., Jung, H.A., Li, H.: Hamiltonism, degree sum and neighborhood intersection. Discrete Math. **90**, 41–52 (1991)
127. Favaron, E., Flandrin, E., Li, H., Liu, Y.P., Tian, F.: Sequences, claws and cyclability. J. Graph Theory **21**, 357–369 (1996)
128. Favaron, O., Flandrin, E., Li, H., Ryjáček, Z.: Clique covering and degree conditions for hamiltonicity in claw-free graphs. Discrete Math. **236**, 65–80 (2001)

129. Fleischner, H., Stiebitz, M.: A solution to a coloring problem. Discrete Math. **101**, 39–48 (1992)
130. Fournier, I.: Thèse d'Etat, LRI, Université de Paris-Sud, France 1985
131. Frieze, A.M.: On matchings and hamilton cycles in random graphs. Surveys in combinatorics, 1989 (Norwich, 1989, pp. 84–114), Lond. Math. Soc. Lect. Notes Ser. **141**, Cambridge: Cambridge University Press 1989
132. Frieze, A.: Hamilton cycles in the union of random permutations. Random Struct. Algorithms **18**, 83–94 (2001)
133. Frieze, A.M., Jerrum, M., Molloy, M., Robinson, R., Wormald, N.: Generating and counting hamilton cycles in random regular graphs. J. Algorithms **21**, 176–198 (1996)
134. Frieze, A., Karoński, M., Thoma, L.: On perfect matchings and hamilton cycles in sums of random trees. SIAM J. Discrete Math. **12**, 208–216 (1999)
135. Frieze, A., Łuczak, T.: Hamiltonian cycles in a class of random graphs: one step further. Random Graphs '87 (Poznanń, 1987, pp. 53–59), Chichester: Wiley 1990
136. Frieze, A., McDiarmid, C., Reed, B.: On a conjecture of Bondy and Fan. Ars Comb. **33**, 329–336 (1992)
137. Gould, R.J.: Updating the hamiltonian problem – a survey. J. Graph Theory **15**, 121–157 (1991)
138. Gould, R.J., Jacobson, M.S., Lesniak, L.: Bipartite versions of two pancyclic results. (preprint)
139. Gould, R.J., Łuczak, T., Pfender, F.: Pancyclicity of 3-connected graphs: pairs of forbidden subgraphs. (preprint)
140. Gould, R.J., Hynds, E.: A note on cycles in 2-factors of line graphs. Bull. Inst. Comb. Appl. **26**, 46–48 (1999)
141. Gould, R.J., Jacobson, M.S.: Forbidden subgraphs and hamiltonian properties of graphs. Discrete Math. **42**, 189–196 (1982)
142. Grunbaum, B., Malkevitch, J.: Pairs of edge-disjoint hamiltonian circuits. Aequationes Math. **14**, 191–196 (1976)
143. Hajnal, A., Szemerédi, E.: Proof of a conjecture of Erdős. In: P. Erdős, A., Renyi and V.T. Sos: Combinatorial Theory and its Applications Vol II, (Colloq. Math. Soc. János Bolyai 4, pp. 601–623) Amsterdam: North Holland 1970
144. Harary, F., Nash-Williams, C. St. J.A.: On eulerian and hamiltonian graphs and line graphs. Can. Math. Bull. 701–710 (1965)
145. Harkat-Benhamdine, A., Li, H., Tian, F.: Cyclability of 3-connected graphs. J. Graph Theory **34**, 191–203 (2000)
146. Hendry, G.R.T.: Extending cycles in graphs. Discrete Math. **85**, 59–72 (1990)
147. Hilbig, F.: Kantenstrukturen in nichthamiltonschen graphen. Ph.D. Thesis, Technische Universität Berlin 1986
148. Hoa, V.D.: Long cycles and neighborhood unions in 1-tough graphs with large degree sums. Discuss. Math. Graph Theory **18**, 5–13 (1998)
149. Hoa, V.D.: A sharp lower bound for the circumference of 1-tough graphs with large degree sums. J. Graph Theory **20**, 137–140 (1995)
150. Hoa, V.D.: Long cycles and neighborhood union in 1-tough graphs with large degree sums. Discuss. Math Graph Theory **18**, 5–13 (1998)
151. Horak, P., Rosa, A.: Decomposing Steiner triple systems in a small configurations. Ars Comb. **26**, 91–105 (1988)
152. Horak, P., Stacho, L.: A lower bound on the number of hamiltonian cycles. Discrete Math. **222**, 275–280 (2000)
153. Hynds, E.: On the structure of 2-factors in line graphs. Ph.D. Thesis, Emory University 2000
154. Jackson, B.: Edge-disjoint hamiltonian cycles in regular graphs of large degree. J. Lond. Math. Soc., Ser. II **19**, 13–16 (1979)
155. Jackson, B.: Hamilton cycles in regular 2-connected graphs. J. Comb. Theory, Ser. B **29**, 27–46 (1980)

156. Jackson, B.: Neighborhood unions and hamiltonian graphs. J. Graph Theory **15**, 443–451 (1991)
157. Jackson, B., Li, H., Zhu, Y.: Dominating cycles in regular 3-connected graphs. Discrete Math. **102**, 163–176 (1991)
158. Jackson, B., Whitty, R.W.: A note concerning graphs with unique f-factors. J. Graph Theory **13**, 577–580 (1989)
159. Jaeger, F.: The 1-factorization of some line graphs. Discrete Math. **43**, 199–206 (1983)
160. Janson, S.: The numbers of spanning trees, hamilton cycles and perfect matchings in a random graph. Comb. Probab. Comput. **3**, 97–126 (1994)
161. Janson, S.: Random regular graphs: asymptotic distributions and contiguity. Comb. Probab. Comput. **4**, 369–405 (1995)
162. Janson, S.: Hamilton cycles in a random tournament. Random Struct. Algorithms **6**, 213–219 (1995)
163. Janson, S., Łuczak, T., Rucinski, A.: Random Graphs. In: Discrete Mathematics and Optimization, New York: John Wiley & Sons 2000
164. Jha, P.K.: Hamiltonian decompositions of products of cycles. Indian J. Pure Appl. Math. **23**, 723–729 (1992)
165. Jung, H.A.: Eine Verallgemeinrung des n-fachen zusammenhangs für Graphen. Math. Ann. **187**, 95–103 (1970)
166. Jung, H.A., Wittmann, P.: Longest cycles in tough graphs. J. Graph Theory **31**, 107–127 (1999)
167. Kaneko, A., Yoshimoto, K.: On a hamiltonian cycle in which specified vertices are uniformly distributed. J. Comb. Theory, Ser. B **81**, 100–109 (2001)
168. Kiel, J.M.: Finding hamiltonian circuits in interval graphs. Inf. Process. Lett. **20**, 201–206 (1985)
169. Kim, J.H., Wormald, N.: Random matchings which induce hamilton cycles and hamiltonian decompositions of random regular graphs. J. Comb. Theory, Ser. B **81**, 20–44 (2001)
170. Kierstead, H., Sárközy, G., Selkow, S.: On k-ordered hamiltonian graphs. J. Graph Theory **32**, 17–25 (1999)
171. Kierstead, H., Quintana, J.: Square hamiltonian cycles in graphs with maximal 4-cliques. Discrete Math. **178**, 81–92 (1998)
172. Komlós, J., Sárközy, G.N., Szemerédi, E.: On the Pósa-Seymour Conjecture. J. Graph Theory **29**, 167–176 (1998)
173. Komlós, J., Sárközy, G.N., Szemerédi, E.: On the square of a hamiltonian cycle in dense graphs. Random Struct. Algorithms **9**, 193–211 (1996)
174. Komlós, J., Sárközy, G.N., Szemerédi, E.: Proof of the Seymour conjecture for large graphs. Ann. Comb. **2**, 43–60 (1998)
175. Komlós, J., Sárközy, G.N., Szemerédi, E.: Proof of the Alon-Yuster conjecture. Discrete Math. **235**, 255–269 (2001)
176. Komlós, J., Sárközy, G.N., Szemerédi, E.: Blow-up lemma. Combinatorica **17**, 109–123 (1997)
177. Komlós, J., Szemerédi, E.: Limit distributions for the existence of hamilton cycles in random graphs. Discrete Math. **43**, 55–63 (1983)
178. Kouider, M.: Covering vertices by cycles. J. Graph Theory **18**, 757–776 (1994)
179. Kouider, M., Lonc, Z.: Covering cycles and k-term degree sums. Combinatorica **16**, 407–412 (1996)
180. Kratsch, D., Lehel, J., Muller, H.: Toughness, hamiltonicity and split graphs. Discrete Math. **150**, 231–245 (1996)
181. Kriesell, M.: A note on hamiltonian cycles in lexicographic products. J. Autom. Lang. Comb. **2**, 135–138 (1997)
182. Kriesell, M.: All 4-connected line graphs of claw-free graphs are hamiltonian connected. J. Comb. Theory, Ser. B **82**, 306–315 (2001)
183. Kriesell, M.: A contribution to a conjecture of A. Saito. (preprint)

184. Lai, H.-J.: Every 4-connected line graph of a planar graph is hamiltonian. Graphs Comb. **10**, 249–253 (1994)
185. Lesniak, L.: Neighborhood unions and graphical properties. Graph theory, combinatorics, and applications. vol. 2 (Kalamazoo, MI, 1988) pp. 783–800, New York: Wiley 1991
186. Li, G.: Edge disjoint hamilton cycles in graphs. J. Graph Theory **35**, 8–20 (2000)
187. Li, H.: Edge disjoint cycles in graphs. J. Graph Theory **13**, 313–322 (1989)
188. Li, H.: Circumferences in 1-tough graphs. Discrete Math. **146**, 145–151 (1995)
189. Li, H., Zhu, Y.J.: Edge-disjoint hamilton cycles in graphs. Graph theory and its applications: East and West (Jian, 1986) (Ann. N. Y. Acad. Sci. 576, pp. 311–322) New York: New York Acad. Sci. 1989
190. Mao-cheng, C., Li, Y., Kano, M.: A $[k, k+1]$-factor containing a given hamiltonian cycle. Electron. J. Comb. **6**, (1999)
191. Martin, P.: Cycles hamiltoniens dans les graphes 4-reguliers 4-connexes. Aequationes Math. **14**, 37–40 (1976)
192. Matthews, M.M., Sumner, D.P.: Longest paths and cycles in $K_{1,3}$-free graphs. J. Graph Theory **9**, 269–277 (1985)
193. Meredith, G.H.J., Lloyd, E.K.: The footballers of Croam. J. Comb. Theory, Ser. B **15**, 161–166 (1973)
194. Mitchem, J., Schmeichel, E.: Pancyclic and bipancyclic graphs – a survey. Graphs and Applications (Boulder, CO 1982) pp. 271–287, New York: Wiley 1985
195. Nash-Williams, C. St. J.A.: Hamiltonian arcs and circuits. Recent Trends in Graph Theory Lect. Notes Math. **185**, pp. 197–210, Springer Verlag 1971
196. Nash-Williams, C. St. J.A.: Combinatorial mathematics and its applications. (Oxford, 1969) pp. 191–200, London: Academic Press
197. Ng, L.: Hamiltonian decompositions of lexicographic products of digraphs. J. Comb. Theory, Ser. B **73**, 119–129 (1998)
198. Ng, L., Schultz, M.: *k*-Ordered hamiltonian graphs. J. Graph Theory **1**, 45–57 (1997)
199. Ore, O.: A note on hamiltonian circuits. Am. Math. Mon. **67**, 55 (1960)
200. Owens, P.J.: Non-hamiltonian maximal planar graphs with high toughness. Cycles and colourings '97 (Statá Lesná) Tatra Mt. Math. Publ. **18**, 89–103 (1999)
201. Ota, K.: Cycles through prescribed vertices with large degree sum. Discrete Math. **145**, 201–210 (1995)
202. Palmer, E.: When do almost all graphs have connectivity exceeding independence number? Graph Theory, combinatorics, and algorithms, vol 1,2 (Kalamazoo, MI, 1992) pp. 851–856, New York: Wiley 1995
203. Pike, D.: Hamilton decompositions of block-intersection graphs of Steiner triple systems. Ars Comb. **51**, 143–148 (1999)
204. Pike, D.: Hamilton decompositions of line graphs of perfectly 1-factorizable graphs of even degree. Aust. J. Comb. **12**, 291–294 (1995)
205. Pike, D.: Hamilton decompositions of some line graphs. J. Graph Theory **20**, 473–479 (1995)
206. Polický, I.: A sufficient condition for hamiltonian graphs. Math. Slovaca **45**, 115–119 (1995)
207. Randerath, B., Schiermeyer, I., Wang, H.: On quadrilaterals in a graph. Discrete Math. **203**, 229–237 (1999)
208. Robinson, R.W., Wormald, N.C.: Almost all cubic graphs are hamiltonian. Random Struct. Algorithms **3**, 117–125 (1992)
209. Robinson, R.W., Wormald, N.C.: Almost all regular graphs are hamiltonian. Random Struct. Algorithms **5**, 363–374 (1994)
210. Robinson, R., Wormald, N.: Hamilton cycles containing randomly selected edges in random regular graphs. Random Struct. Algorithms **19**, 128–147 (2001)
211. Rosenfeld, M.: Pairs of edge disjoint hamiltonian circuits in 5-connected planar graphs. Aequationes Math. **38**, 50–55 (1989)

212. Ryjáček, Z.: On a closure concept in claw-free graphs. J. Comb. Theory, Ser. B **70**, 217–224 (1997)
213. Ryjáček, Z., Saito, A., Schelp, R.: Claw-free graphs with complete closure. (preprint)
214. Ryjáček, Z., Schelp, R.H.: Contractability techniques as a closure concept. (preprint)
215. Saad, R.: Finding a longest alternating cycle in a 2-edge coloured complete graph is in RP. Comb. Probab. Comput. **5**, 297–306 (1996)
216. Sauer, N., Spencer, J.: Edge disjoint placement of graphs. J. Comb. Theory, Ser. B **25**, 295–302 (1978)
217. Stacho, L.: Old hamiltonian ideas from a new point of view. Math. Nachr. **186**, 251–258 (1997)
218. Stacho, L.: A sufficient condition guaranteeing large cycles in graphs. Discrete Math. **169**, 273–277 (1997)
219. Stacho, L.: Cycles through specified vertices in 1-tough graphs. Ars Comb. **56**, 263–269 (2000)
220. Schmeichel, E., Hakimi, S.L.: A cycle structure theorem for hamiltonian graphs. J. Comb. Theory, Ser. B **45**, 99–107 (1988)
221. Seymour, P.: Problem section in Combinatorics: In: T.P. McDonough and V.C. Mavron: Proc. British Combinatorial Conference 1973, pp. 201–202, Cambridge University Press 1974
222. Shearer, J.: A property of colored complete graphs. Discrete Math. **25**, 175–178 (1979)
223. Sheehan, J.: The multiplicity of hamiltonian circuits in a graph. In: M. Fiedler: Recent Advances in Graph Theory, pp. 447–480, Prague: Academia 1975
224. Shepherd, F.B.: Hamiltonicity in claw-free graphs. J. Comb. Theory, Ser. B **53**, 173–194 (1991)
225. Shi, R.: The Ore-type conditions on pancyclism of hamiltonian graphs. J. Syst. Sci. Math. Sci. **11**, 79–90 (1991)
226. Shi, R.: 2-Neighborhoods and hamiltonian conditions. J. Graph Theory **16**, 267–271 (1992)
227. Song, Z.M.: Degrees, neighborhood unions and hamiltonian properties. Ars Comb. **34**, 205–211 (1992)
228. Song, Z.M., Zhang, K.M.: Neighborhood unions and hamiltonian properties. Discrete Math. **133**, 319–324 (1994)
229. Stong, R.: Hamilton decompositions of cartesian products of graphs. Discrete Math. **90**, 169–190 (1991)
230. Szemerédi, E.: Regular partitions of graphs. Colloq. Int. CNRS no. 260, pp. 399–401, Problemes Combinatoires et Theorie des Graphes, Orsay 1976
231. Thomason, A.G.: Hamiltonian cycles and uniquely edge colourable graphs. Ann. Discrete Math. **3**, 259–268 (1978)
232. Thomassen, C.: Reflections on graph theory. J. Graph Theory **10**, 309–324 (1986)
233. Thomassen, C.: Independent dominating sets and a second hamiltonian cycle in regular graphs. J. Comb. Theory, Ser. B **72**, 104–109 (1998)
234. Thomassen, C.: Chords of longest cycles in cubic graphs. J. Comb. Theory, Ser. B **71**, 211–214 (1997)
235. Thomassen, C.: On the number of hamiltonian cycles in bipartite graphs. Comb. Probab. Comput. **5**, 437–442 (1996)
236. Tian, F., Zang, W.: Bipancyclism in hamiltonian bipartite graphs. J. Syst. Sci. Math. Sci. (English Series) **2**, 22–31 (1989)
237. Tutte, W. T.: A theorem on planar graphs. Trans. Am. Math. Soc. **82**, 99–116 (1956)
238. Tutte, W. T.: On hamiltonian circuits. J. Lond. Math. Soc. **21**, 98–101 (1946)
239. van Blanken, E., van den Heuvel, J., Veldman, H.J.: Pancyclicity of hamiltonian line graphs. Discrete Math. **138**, 379–385 (1995)
240. Wei, B.: Hamiltonian cycles in 1-tough graphs. Graphs Comb. **12**, 385–395 (1996)
241. Wei, B., Zhu, Y.: Hamiltonian *k*-factors in graphs. J. Graph Theory **25**, 217–227 (1997)

242. Witte, D., Gallian, J.A.: A survey-Hamiltonian cycles in Cayley graphs. Discrete Math. **51**, 293–304 (1984)
243. Xiong, L.: Edge degree conditions for subpancyclicity in line graphs. Discrete Math. **188**, 225–232 (1998)
244. Zaks, J.: Pairs of hamiltonian circuits in 5-connected planar graphs. J. Comb. Theory, Ser. B **21**, 116–131 (1976)
245. Zhan, S.: Hamiltonian connectedness of line graphs. Ars Comb. **22**, 89–95 (1986)
246. Zhang, S.M.: Pancyclism and bipancyclism of hamiltonian graphs. J. Comb. Theory, Ser. B **60**, 159–168 (1994)
247. Zhang, S., Li, X., Broersma, H.: Heavy paths and cycles in weighted graphs. Discrete Math. **223**, 327–336 (2000)
248. Zhou, M.: Decomposition of some product graphs into 1-factors and hamiltonian cycles. Ars Comb. **28**, 258–268 (1989)

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