All work on this assessment should be your own. The technology allowed on this test includes: Desmos (`https://www.desmos.com/calculator`) and an approved TI calculator. This exam has 10 questions for a total of 100 points.

1. (9 points) Use the graph shown in Figure 1 to answer the following questions about limits of a graph.

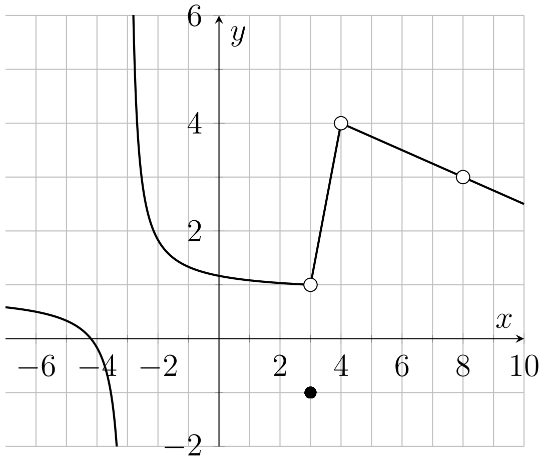


Figure 1: Graph of $f(x)$

(a) $\lim\limits_{x\to -3^-} f(x) =$______ (d) $\lim\limits_{x\to 3^+} f(x) =$______ (g) $\lim\limits_{x\to 8^+} f(x) =$______

(b) $\lim\limits_{x\to -3^+} f(x) =$______ (e) $f(3) =$______ (h) $\lim\limits_{x\to 8^-} f(x) =$______

(c) $\lim\limits_{x\to 3^-} f(x) =$______ (f) $\lim\limits_{x\to 3} f(x) =$______ (i) $f(8) =$______

2. (4 points) Find the equation of the line tangent to $f(x) = 3x^3 + x^2 - 4x + 9$ at $x = 1$.

3. (16 points) *Choose four.* Use algebra and limit rules to calculate **four** of the following five limits. Some limits may be made easier by L'Hôpital's rule and some may not. Circle which four you want graded. There is no extra credit for doing five limits. You may use graphs to verify your answers, but that cannot be your sole reason for your answer. *You must provide an algebraic defense of your answer in order to receive full credit.*

(a) $\displaystyle\lim_{x\to -5} \frac{x^2-25}{x+5}$

(b) $\displaystyle\lim_{x\to 1} \frac{\sin(\ln x)}{x-1}$

(c) $\displaystyle\lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h}$

(d) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{\tan x}$

(e) $\displaystyle\lim_{x\to\infty} 2x^{-\frac{4}{3}}$

4. (20 points) *Choose four.* Use algebra and derivative rules to calculate the derivative (with respect to x) of **four** of the following five functions. Circle which four you want graded. There is no extra credit for doing five derivatives. Please do not simplify.

(a) $f(x) = 4x^2 e^{2x}$

(b) $g(x) = \displaystyle\int_{-\pi}^{\cos(x)} \left(\frac{e^t \ln t}{t^2 - 1}\right) dt$

(c) $h(x) = \dfrac{\sin(3x+1)}{\sqrt{x}}$

(d) $k(x) = \displaystyle\int_1^{x^2} \tan\left(\ln(t)\right)\, dt$

(e) $m(x) = \ln(x)(x^2 - 4x)^2$

5. (8 points) Let $h(x) = x^2e^x - 1$.

   (a) Fill in the sign charts for $h(x)$. *You may use approximate values (i.e. decimal approximations) of the values for h''.*

$\longleftrightarrow h'$ $\qquad\qquad$ $\longleftrightarrow h''$

Sign Charts of h

   (b) Critical Point(s) ____________

   (c) Increasing Interval ____________

   (d) Decreasing Interval ____________

   (e) Inflection point(s) ____________

   (f) Concave Up on ____________

   (g) Concave Down on ____________

6. Let $g(x) = e^{-x^2}$.

   (a) (3 points) Show that $g(x)$ has a maximum value of 1 using calculus.

   (b) (4 points) Find $g''(x)$.

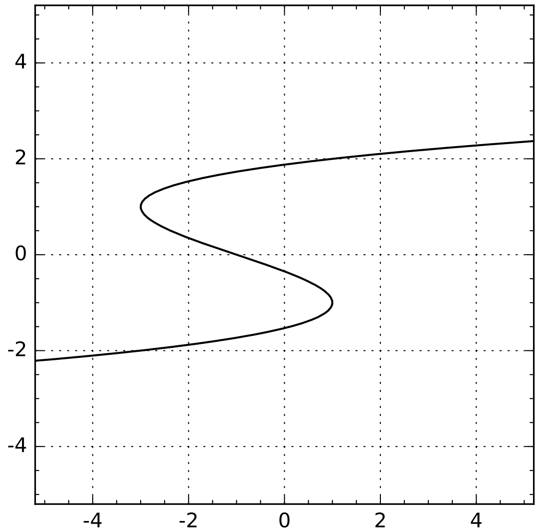
   (c) (2 points) Use your second derivative to find the x-value where $g(x)$ is increasing the fastest?

   (c) ____________

   (d) (2 points) Use your second derivative to find the x-value where $g(x)$ is decreasing the fastest?

   (d) ____________

7. (6 points) Consider the graph of the solutions of the equation $y^3 - 3y - x = 1$ shown below.



(a) Find $\dfrac{dy}{dx}$.

(b) Find the slope of each point where $x = -3$ on the graph above.

8. (16 points) *Choose four.* Use algebra, integration formula, trigonometric functions, realizing that the integrand is a derivative of a product/quotient, educated guess-and-check, and/or u-substitution strategies to calculate all four of the following indefinite integrals.

(a) $\displaystyle\int \left(4x^3 - 6x^2 + 2x - 7\right) dx$

(b) $\displaystyle\int \left(5x^7 e^{5x} + 7x^6 e^{5x}\right) dx$

(c) $\displaystyle\int \frac{\sqrt{\ln x}}{x} dx$

(d) $\displaystyle\int (e - \pi) dx$

9. (6 points) *Choose 2.* Use the Fundamental Theorem of Calculus to determine the *exact* value of **two** of the following definite integrals. Decimal form will receive half credit.

(a) $\displaystyle\int_1^4 \frac{2x}{\sqrt{x^2+1}}dx$

(b) $\displaystyle\int_0^1 2^x dx$

(c) $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}dx$

10. (4 points) Let $g(x) = x^2 + 3$.

(a) Find the average value of $g(x)$ on the interval $[1, 4]$.

(b) Find the exact value of c such that $g(c)$ is equal to the average value. In other words, apply the Mean Value Theorem for Integrals to the function $g(x)$.