Advanced Quantitative methods

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Elementary probability theory

Some probability rules : multiplication.

To start with, consider the following

Examples

- You roll two dice. What is the probability to get a double 5?
- Have a collection of 6 balls on a bag (3 green, 2 blue, 1 red). You pick two balls randomly, without replacing the first. What is the probability to get 2 green balls?

Some considerations

- We want P(5 on 1st dice **and** on 2nd dice).
- We want P(ball 1 is green **and** ball 2 is green).

Similiarities and differences

- In both cases we have **compund** events
  - Dice example: the result of the 1st dice does not affect the probability for the 2nd dice.
  - Ball problem, the colour of the first ball does affect the probability to get a green ball on the 2nd draw (since we do not put the 1st ball back).

Definition

Two events are **independent** if the occurrence or non occurrence of one of them does not change the probability that the other event will occur.

Multiplication rule

If two events A and B are **independent**, then

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

What about **dependent** events?

- Need to take into account the changes in the probability of one event caused by the occurrence of the other event.
- We denote by $P(A|B)$ the probability of A, given that B takes place. It is a **conditional probability**

Multiplication rule (dependent events)

$$P(A \text{ and } B) = P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$$

- Note that the two formulas compute the same thing, the difference is **on what we are conditioning**

Example (the balls problem, solved)

More examples

- Guided Exercise n.4
- Guided Exercise n.5

Addition rules

Another kind of combination

Consider the possibility if having one event or another occurring.

Example

A sports car saleswoman gets a bonus if she sells a convertible or a car with leather upholstery. She will be interested in the probability that a client buys a car that is either convertible or has leather upholstery.

- A convertible car with leather upholstery also works (but the bonus is the same)

To retain :

The condition "A or B" is satisfied by any of the results:

- an outcome from A,
- an outcome from B,
- an outcome from both A and B.

Do guided exercise n. 6

Example 6

One has an introductory statistics class with 31 students, distributed as follows:

Gender	Freshmen	Sophomore	Junior	Senior
Female	9	3	4	1
Male	6	5	2	1

We choose, randomly, a student. Give the following probabilities:

1. The student is either freshman or sophomore,
2. The student is either a male or a sophomore.

Definition

Two events A and B are **mutually exclusive** (or **disjoint**) if they cannot occur together. Equivalently $P(A \text{ and } B) = 0$.

Addition rule

For any two events A and B we have :

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Remarks

- If the events are disjoint, one simply adds,
- If the events are not disjoint, one needs to subtract.
- The same happens for more than 2 events.

Example (ex n.7 from the book)

Laura rolls two dice. She wants to obtain a result which is bigger than 8. What is the probability she gets the result she wants?

Solution

On the board

Example (ex n.8 from the book)

At Hopewell Electronics, all employees were asked about their political affiliations. The employees were grouped by type of work, as executives or production workers. The results with row and column totals are shown below

Type	Democrat (D)	Rep. (R)	Ind. (I)	Row
Executive (E)	5	34	9	48
Prod. worker (PW)	63	21	8	92
Column	68	55	17	140

Determine

1. $P(D)$ and $P(E)$,
2. $P(D|E)$,
3. Are the events D and E independent?
4. Determine $P(D \text{ and } E)$
5. Determine $P(D \text{ or } E)$