1. (6 points) Explain why if AB and BA are both defined (i.e. you can calculate both of them), then AB and BA are both square matrices.

2. (6 points) Explain why it is not necessarily true that if $A, B \in \mathbb{R}^{n\times n}$, then $(A - B)^2 = A^2 - B^2$.

3. (6 points) If A is a matrix in $\mathbb{R}^{2\times 2}$, how can you quickly tell if it is invertible WITHOUT performing any row operations?

4. (6 points) Find the LU factorization of the following matrix:

$$\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}.$$

5. Consider the following sequence of row operations.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ -1 & 1 & -1 \end{bmatrix} \underset{\sim}{\text{Op. 1}} \begin{bmatrix} 2 & -2 & 2 \\ 0 & 1 & 0 \\ -1 & 1 & -1 \end{bmatrix} \underset{\sim}{\text{Op. 2}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\underset{\sim}{\text{Op. 3}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underset{\sim}{\text{Op. 4}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) (16 points) Use the operations defined above to fill in the elementary operation and the corresponding elementary matrix that completes the operation. Yes, you should consider a row swap a elementary row operation for this problem.

Operation:	Elementary Operation:	Elementary Matrix:
1		$E_1 = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$
2		$E_2 = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$
3		$E_3 = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$
4		$E_4 = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

(b) (4 points) Does A^{-1} exist? Defend your answer using the information from the table.

(c) (2 points) If A is the coefficient matrix of a system of linear equations with constant vector $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, then the system is A. consistent with a unique solution. B. consistent with infinitely many solutions. C. inconsistent.

(c) ______________

6. (15 points) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$, $\vec{b}_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\vec{b}_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. If $A^{-1} = \begin{bmatrix} 3 & -1 \\ -1 & \frac{1}{2} \end{bmatrix}$.

Use A^{-1} to solve the three systems

$$A\vec{x} = \vec{b}_1, \quad A\vec{x} = \vec{b}_2, \quad A\vec{x} = \vec{b}_3.$$

7. Below is the LU Factorization of a matrix.

$$C = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 6 & -4 & 5 & -3 \\ 8 & -4 & 1 & 0 \\ 4 & -1 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 2 & -1 & 5 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

(a) (5 points) What does the -1 in the L matrix encode about the process of elimination that was completed?

(b) (4 points) Using the LU Factorization, what is the determinant of the matrix?

(b) ____________

(c) (4 points) Suppose the 2 in the first column of matrix C was zero. What can (or cannot) be said about the LU factorization of C in this case?

8. (16 points) Circle (or box) TRUE or FALSE to indicate if the statement is true or false. Each question is worth two points.

(a) TRUE / FALSE

For any matrix, both AA^T and A^TA are defined.

(b) TRUE / FALSE

If A, B and X are invertible matrices such that $XA = B$, then $X = A^{-1}B$.

(c) TRUE / FALSE

The inverse of an elementary matrix is also an elementary matrix.

(d) TRUE / FALSE

If $BC = BD$, then $C = D$.

(e) TRUE / FALSE

If A is invertible and $r \neq 0$, then $(rA)^{-1} = r(A)^{-1}$.

(f) TRUE / FALSE

Suppose $B \overset{\text{RREF}}{\longrightarrow} I$. Then there exists some number of elementary matrices such that $E_pE_{p-1}\cdots E_2E_1A = I$.

(g) TRUE / FALSE

Any matrix in $\mathbb{R}^{4\times 4}$ requires AT MOST 6 row operations in order to upper triangularized.

(h) TRUE / FALSE

If E is an elementary matrix, then E^T is also an elementary matrix.