All work on this assessment should be your own. The technology allowed on this test includes: Desmos (https://www.desmos.com/calculator) and an approved TI calculator. This exam has 11 questions, for a total of 100 points.

This exam is due Wednesday, April $11^{\text {th }}, 2018$ by 1:00 PM

1. The following questions are on Rolle's Theorem and the Mean Value Theorem.
(a) (4 points) Let $f(x)=\sqrt{x(1-x)}$. In detail, determine if $f$ satisfies the hypotheses of Rolle's Theorem on the interval $[0,1]$. If yes, use derivatives and algebra to find the exact value of all $c \in(0,1)$ that satisfies the conclusion of Rolle's Theorem. If not, explain why. If not, explain why. Your answer should be in exact for, i.e. no decimals.
$\square$
(b) (4 points) Let $g(x)=\arcsin (x)$. In detail, determine if $g$ satisfy the hypotheses of the Mean Value Theorem on the interval $[-1,1]$. If yes, use derivatives and algebra to find the exact value of all $c \in(-1,1)$ that satisfy the conclusion of the Mean Value Theorem. If not, explain why. Your answer should be in exact for, i.e. no decimals.
2. (6 points) On each sign chart, unlabeled tick marks are locations where $f^{\prime}(x)$ or $f^{\prime \prime}(x)$ is zero and $x$-values where $f^{\prime}(x)$ or $f^{\prime \prime}(x)$ does not exist are indicated by tick-marks labeled "DNE". Sketch a graph of $f^{\prime}$ and $f$.


(a) Graph of $f^{\prime}$
(b) Graph of $f$
3. (6 points) Below is the graph of a function, $h(x)$. Create the first and second derivative sign chart of the function. A larger, more detailed, picture can be accessed at https://goo.gl/khx78g.


Figure 2: Graph of $h(x)$


Sign Charts of $h$
4. (18 points) Let $g$ the function $g(t)=\frac{4 x}{x^{2}+4}$, also known at Newton's serpentine. Create two sign charts: one for the first derivative and one for the second derivative. Once you have completed the sign charts, use them to find the characteristics of the function. Answers that are points should have both an $x$ and a $y$ value and written as a coordinate. Exact answers are perfered, but not required here.


## Sign Charts of $g$

- domain: $\qquad$ - concave up: $\qquad$
- range: $\qquad$ - concave down: $\qquad$
- $x$-intercept(s): $\qquad$ - local minima: $\qquad$
- $y$-intercept(s): $\qquad$ - local maxima: $\qquad$
- critical point(s): $\qquad$ - global minima: $\qquad$
- inflection point(s): $\qquad$ - global maxima: $\qquad$
- inc. interval: $\qquad$ $-\lim _{t \rightarrow \infty} g(t)=$ $\qquad$
- dec. interval: $\qquad$ $-\lim _{t \rightarrow \infty} g(t)=$ $\qquad$

5. (6 points) Suppose that $f$ and $f^{\prime}$ are continuous functions, that $f(0)=0$, and that

$$
\lim _{x \rightarrow 0} \frac{f(x)}{\sin (2 x)}=5
$$

Find $f^{\prime}(0)$.
6. (5 points) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius $R$. [Hint: the volume of a cone is $V=\frac{1}{3} \pi r^{2} h$.]

$\square$
7. (5 points) An object attached to a spring, in the presents of friction, undergoes "damped" harmonic motion. The equation for damped harmonic motion is

$$
x(t)=A e^{-\frac{b}{2 m} t} \cos (\omega t+\phi),
$$

where $A$ is the amplitude, $b$ is the damping constant or drag coefficient, $m$ is the mass of the object attached to the spring, $\omega$ is the angular frequency in radians per second, $\phi$ is a constant phase shift in radians. Suppose an object is displaced 6 inches from the spring's equilibrium, has a drag coefficient of 1 , a mass of 6 grams, an angular frequency of 1 radian per second, and a 0 radians phase shift. Show that the maximum displacement of the system occurs at $t=\pi n+\operatorname{arccot}(-12)$ for all $n \in \mathbb{Z}_{\geq 0}$.
8. (12 points) A person in a boat is 2 miles from the nearest point on the coast. He is trying to go to point $Q$, located 2 miles down the coast and 3 miles inland. He can row at 1 mile per hour and jog at 7 miles per hour. You will need to assume that the shoreline matches perfectly with the $y$ axis in the illustration.

(a) Write a formula for the time it takes the individual to travel from their boat to point $Q$ as a function of $x$ and call it $T(x)$. [Recall: $d=v \cdot t$.]
(b) Find $\frac{d T}{d x}$.
$\square$
(c) Use a graphing utility to find the $x$ value that minimizes the time traveled. Round your answer to three decimal places. Don't forget your units.
(c) $\qquad$
(d) How long did it take the individual to get from their boat to point $Q$ ? Round your answer to three decimal places. Don't forget your units.
(d) $\qquad$
9. (8 points) A cone with radius of 2 feet and height of 4 feet is being filled with water.

(a) Use similar triangles to write the radius as a function of the height.
(b) Use the information from part (a) to write a formula for the volume of the water in the tank as a function of only the height.
$\square$
(c) If the height is changing at a constant rate of 2 feet per second, at what rate is the volume changing when the water is 1 foot deep in the cone? Write your answer in exact form, i.e. no decimals.
10. (16 points) Use algebra and/or L'Hôspital's rule to calculate the five limits. Some limits may be made easier by L'Hôpital's rule and some may not. You may use graphs to verify your answers, but that cannot be your sole reason for your answer. Answers without calculus/algebra to justify your answer will be counted incorrect.
(a) $\lim _{x \rightarrow 1} \frac{x-1}{e^{x-1}-1}$
(b) $\lim _{x \rightarrow \infty} x\left(\frac{1}{2}\right)^{x}$
$\qquad$
(c) $\lim _{x \rightarrow 0} \frac{\sin x}{x+\sin x}$
(d) $\lim _{x \rightarrow 1} \frac{\sin (\ln x)}{x-1}$
$\square$
(e) $\lim _{x \rightarrow \infty} x^{\frac{1}{x}}$
$\square$
11. (10 points) Circle (or box) TRUE or FALSE to indicate if the statement is true or false. If you answer false, you must provide a counterexample or a reason why the statement is false. A counterexample can take several forms such as a graph or a particular example where the statement is false. Each question is worth two points.
(a) TRUE / FALSE

If $f$ is continuous and differentiable on $[0,10]$ with $f^{\prime}(x)=0$, then $f$ has a local maximum or minimum at $x=5$.
(b) True / False

If $f$ is concave down on an interval $I$, then $f^{\prime \prime}$ is positive on that interval.
(c) TRUE / FALSE

If $f^{\prime}(1)$ is negative and $f^{\prime}(3)$ is positive, then $f$ has a local minimum at $x=2$.
(d) TRUE / FALSE

Suppose the ratio of a cone's radius to its height is growing at a constant rate. Then the volume of the come will also increase at a constant rate.
(e) TRUE / FALSE

If $\lim _{x \rightarrow 2} \ln (f(x))=4$, then $\lim _{x \rightarrow 2}(f(x))=\ln (4)$

