## Physics 211 Formula Sheet

## E1

$\left|\vec{F}_{e}\right|=k \frac{\left|q_{1} q_{2}\right|}{r^{2}}$
$\vec{F}_{e}=q \vec{E}$
$\vec{E}=k \frac{Q}{\left|\vec{r}_{P S}\right|^{2}} \hat{r}_{P S}$
$\vec{F}_{e}$ electrostatic force vector
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$q_{1}, q_{2}$ charge of particles
$r$ distance between particles
$\vec{F}_{e}$ electrostatic force vector
$q$ charge of test particle
$\vec{E}$ electric field vector
$\vec{E}$ Electric field vector at a point $P$ due to a single particle
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$Q$ charge of source particle
$\vec{r}_{P S}$ position of $P$ relative to source particle
$\hat{r}_{P S}$ unit vector in the direction of $\vec{r}_{P S}$

E2
$\vec{E}=k \frac{2 \vec{p}_{e}}{\left|\vec{r}_{P D}\right|^{3}}$
$\left|\vec{p}_{e}\right|=q s$
$\vec{E}=2 k \frac{\lambda}{\left|\vec{r}_{P N}\right|} \hat{r}_{P N}$
$\vec{E}=(2 \pi k) \sigma \hat{r}_{P N}=\frac{\sigma}{2 \epsilon_{0}} \hat{r}_{P N}$
$\vec{E}$ Electric field vector at a distant point $P$ along a dipole's axis
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$\vec{p}_{e}$ dipole moment vector
$\vec{r}_{D P}$ position of $P$ relative to dipole's center
$\vec{p}_{e}$ dipole moment vector
$q$ charge of diploe's positive particle
$s$ particle's separation
$\vec{E}$ Electric field vector at a point $P$ by an infinite linear charge dist.
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$\lambda$ linear charge density
$\vec{r}_{P N}$ position of $P$ relative to the nearest point on the line
$\hat{r}_{P N}$ unit vector in the direction of $\vec{r}_{P N}$, perpendicularly away from the line
$\vec{E}$ Electric field vector at a point $P$ by an infinite planar charge dist.
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$\sigma$ charge per unit area
$\hat{r}_{P N}$ unit vector points perpendicularly away from the plane, at $P$ $\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}$

$$
\phi(x, y, z)=\frac{V_{e}(x, y, z)}{q}
$$

$$
\phi=k \frac{Q}{\left|\vec{r}_{P S}\right|}
$$

$$
\Delta \phi=-\int \vec{E} \cdot d \vec{r}
$$

$$
\vec{E}(x, y, z)=\left[\begin{array}{l}
E_{x} \\
E_{y} \\
E_{z}
\end{array}\right]=\left[\begin{array}{l}
-\partial \phi / \partial x \\
-\partial \phi / \partial y \\
-\partial \phi / \partial z
\end{array}\right]=-\vec{\nabla} \phi
$$

$\phi$ electric potential at a point with coordinates $x, y, z$
$V_{e}$ total electrostatic potential energy
$q$ charge of test particle
$\phi$ electric potential
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$Q$ charge of fixed particle
$\vec{r}_{p s}$ position of the observation point, $P$, relative to the fixed particle
$\Delta \phi$ difference in potential between two points
$\vec{E}$ Electric field vector
$d \vec{r}$ step between the two points
$\vec{E}$ Electric field vector at a point with coordinates $x, y, z$ $\phi$ electric potential at a point with coordinates $x, y, z$

E4
$C \equiv\left|\frac{Q}{\Delta \phi}\right|=\epsilon_{0} \frac{A}{S}$

## $C$ Capacitance

$Q$ charge of either plate of a capacitor
$\Delta \phi$ difference in potential between the two plates
$A$ Area of one plate
$s$ separation of the plates
$\epsilon_{0}=8.85 \times 10^{-12}$
$u_{E}=\frac{1}{2} \epsilon_{0}|\vec{E}|^{2}=\frac{1}{8 \pi k}|\vec{E}|^{2}$
$U=\frac{1}{2} C(\Delta \phi)^{2}$
$u_{E}$ energy density of an electric field
$\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}$
$k=8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}$
$\vec{E}$ Electric field vector
$U$ energy stored in the electric field of a capacitor
$C$ capacitance of the capacitor
$\Delta \phi$ difference in potential between the two plates

| $\vec{v}_{d}=-\frac{e}{m_{e}} \vec{E} \tau$ | $\vec{v}_{d}$ drift velocity of an electron $e$ elementary charge |
| :---: | :---: |
|  | $m_{e}$ mass of an electron <br> $\vec{E}$ Electric field vector <br> $\tau$ time between collisions |
|  | $\vec{J}$ current density in the neighborhood of a point in a conductor $\rho$ charge per unit volume |
| $\vec{J}=\rho \vec{v}_{d}=n q \vec{v}_{d}$ | $\vec{v}_{d}$ drift velocity <br> $n$ number density (number of carriers per unit volume) <br> $q$ charge of each carrier |
| $\vec{J}=\sigma_{c} \vec{E}$ | $\vec{J}$ current density in the neighborhood of a point in a conductor $\sigma_{c}$ conductivity <br> $\vec{E}$ Electric field at that point |
| $\sigma_{c}=\frac{n q^{2} \tau}{m}$ | $\sigma_{c}$ conductivity $n$ number density |
|  | $q$ charge of each carrier <br> $\tau$ time between collisions <br> $m$ mass of a carrier |
| $n=N_{1} \rho_{m} \frac{N_{A}}{M_{A}}$ | $n$ number density <br> $N_{1}$ number of electrons per one atom |
|  | $\rho_{m}$ metal's mass density <br> $N_{A}$ Avagadro's number, $6.02 \times 10^{23}$ <br> $M_{A}$ atomic weight |
| $\vec{J}_{N}=n \vec{v}$ | $\vec{J}_{N}$ number current density |
|  | $\vec{v}$ velocity <br> $n$ number density |
| $\frac{\Delta N}{\Delta t}=\vec{J}_{N} \cdot \vec{A}$ | $\Delta N$ change in the number of particles $\Delta t$ change in time |
|  | $\vec{J}_{N}$ number current density $\vec{A}$ area Vector |
|  | $i$ current |
| $i \equiv \int_{S} \vec{J} \cdot d \vec{A}$ | $S$ surface <br> $\vec{J}$ current density <br> $\vec{A}$ tile vector |

$$
\left|\Delta \phi_{\text {wire }}\right|=|\vec{E}| L
$$

$$
R=\frac{|\Delta \phi|}{I}
$$

$$
R=\frac{L}{\sigma_{c} A}
$$

$$
P=I|\Delta \phi|=\frac{|\Delta \phi|^{2}}{R}=I^{2} R
$$

$I(t)=I_{0} e^{-t / R C}$
$|\Delta \phi(t)|=\left|\Delta \phi_{0}\right| e^{-t / R C}$
$Q(t)=Q_{0} e^{-t / R C}$
$\Delta \phi_{\text {wire }}$ potential difference between the ends of a wire
$\vec{E}$ Electric field inside the wire
$L$ Length of wire
$R$ Resistance
$|\Delta \phi|$ potential difference
$I$ current
$R$ Resistance of wire
$\mid L$ length of wire
$A$ Cross sectional area $\sigma_{c}$ resistivity of wire
$P$ Power dissipated, $R$ Resistance
$|\Delta \phi|$ potential difference, $I$ current
$I$ current flowing through the circuit
$|\Delta \phi|$ potential difference between the plates $Q$ charge on the capacitor's positive plate
$I_{0}=I(0), Q_{0}=Q(0),\left|\Delta \phi_{0}\right|=|\Delta \phi(0)|$
$e$ euler's constant $\approx 2.7183, t$ time
$R$ Resistance
$C$ Capacitance of capacitor
$R_{\text {set }}$ equivalent resistance of a set of resistors in series $R_{n}$ resistance of the $n$th resistor
$R_{\text {set }}$ equivalent resistance of a set of resistors in parallel
$R_{n}$ resistance of the $n$th resistor
E8
$\vec{F}_{m}$ magnetic force on a charged particle
$q$ charge of particle
$\vec{v}$ velocity of particle
$\vec{B}$ magnetic field
$R$ radius of helical motion
$\vec{p}_{\perp}$ component of the particle's momentum perpendicular to $\vec{B}$
$q$ charge of particle
$\vec{B}$ magnetic field
$T$ period of helical motion
$m$ mass of particle
$\vec{B}$ magnetic field
$q$ charge of particle

$$
\begin{aligned}
& \vec{F}_{m, \text { seg }} \text { force on a segment of wire } \\
& L \text { length of segment of wire } \\
& \vec{I} \text { current vector } \\
& \vec{B} \text { magnetic field } \\
& \vec{\mu} \text { magnetic moment vector } \\
& N \text { number of turns } \\
& I \text { current } \\
& \vec{A} \text { tile vector of loop } \\
& \vec{\tau} \text { torque on a loop } \\
& \vec{\mu} \text { magnetic moment vector } \\
& \vec{B} \text { magnetic field } \\
& V_{m} \text { magnetic potential energy } \\
& \vec{\mu} \text { magnetic moment vector } \\
& \vec{B} \text { magnetic field } \\
& P \text { power output of an electric motor } \\
& \vec{\mu} \text { magnetic moment vector } \\
& \vec{B} \text { magnetic field } \\
& f \text { frequency of motor (turns per second) } \\
& \Delta \phi \text { potential difference between a bar's ends } \\
& \vec{v} \text { velocity of bar } \\
& \vec{B} \text { magnetic field } \\
& L \text { length of bar } \\
& W \text { Work by a spacially changing magnetic field on a charge carrier in a loop } \\
& q \text { charge of a carrier } \\
& \vec{v} \text { velocity of bar } \\
& \Delta \vec{B} \text { change in magnetic field } \\
& L \text { length of leg of loop }
\end{aligned}
$$

$$
\vec{B}=\frac{\mu_{0}}{4 \pi} \frac{q}{\left|\vec{r}_{P S}\right|^{2}}\left(\vec{v} \times \hat{r}_{P S}\right)
$$

$$
\vec{B}=\frac{\mu_{0}}{4 \pi} \sum_{\text {all } \mathrm{i}} \frac{d L}{\left|\vec{r}_{P i}\right|^{2}}\left(\vec{I}_{i} \times \hat{r}_{P i}\right)
$$

$d L$ length of the $i$ th segment of wire
$\vec{I}_{i}$ Current flowing through the $i$ th segment of wire
$\vec{r}_{P i}$ position vector of $P$ relative to the charged particle
$\hat{r}_{P i}$ unit vector of the in the direction of $\vec{r}_{P i}$
$\vec{B}$ Magnetic field produced by an infinite, straight wire (with cst. current)
$\mu_{0}$ magnetic permeability, $\mu_{0} / 4 \pi=10^{-7} \mathrm{Tsm} / \mathrm{C}$
$I$ constant current
$r$ distance to the nearest point of the wire
$\vec{B}$ Magnetic field produced at the center of a circular loop (with cst. current) $\mu_{0}$ magnetic permeability, $\mu_{0} / 4 \pi=10^{-7} \mathrm{Tsm} / \mathrm{C}$
$|\vec{B}|=\frac{\mu_{0} I}{2 R}$

## E11

$$
\vec{F}_{e m}=q(\vec{E}+\vec{v} \times \vec{B})
$$

$\vec{F}_{e m}$ Electromagnetic force
$q$ charge of moving particle
$\vec{E}$ Electromagnetic field
$\vec{v}$ velocity of moving particle
$\vec{B}$ magnetic field

$$
\left[\begin{array}{c}
E_{x}^{\prime} \\
E_{y}^{\prime} \\
E_{z}^{\prime}
\end{array}\right]=\left[\begin{array}{c}
E_{x} \\
\gamma\left(E_{y}-\beta B_{z}\right) \\
\gamma\left(E_{z}+\beta B_{y}\right)
\end{array}\right]
$$

$q$ charge of particle
$\vec{v}$ velocity of the particle
$\vec{r}_{P S}$ position vector of $P$ relative to the charged particle
$\hat{r}_{P S}$ unit vector of the in the direction of $\vec{r}_{P S}$
$\vec{B}$ magnetic field vector created at point $P$ by a uniform, arbitrarily shaped wire

$$
\mu_{0} \text { magnetic permeability, } \mu_{0} / 4 \pi=10^{-7} \mathrm{Tsm} / \mathrm{C}
$$

$|\vec{B}|=\frac{\mu_{0} I r}{4 \pi} \frac{2}{r^{2}}=\frac{\mu_{0} I}{2 \pi r}$
$I$ constant current
$R$ Radius of loop
$\vec{E}$ Electric field in the home frame
$\vec{B}$ Magnetic field in the home frame
$\vec{E}^{\prime}$ Electric field in the prime frame
$\vec{B}^{\prime}$ Magnetic field in the prime frame

$$
\left[\begin{array}{c}
\not B_{x}^{\prime} \\
B_{y}^{\prime} \\
B_{z}^{\prime}
\end{array}\right]=\left[\begin{array}{c}
\not B_{x} \\
\gamma\left(\not B_{y}+\beta E_{z}\right) \\
\gamma\left(\not B_{z}-\beta E_{y}\right)
\end{array}\right]
$$

$\beta c$ velocity of prime frame relative to home frame
$\gamma \equiv\left(1-\beta^{2}\right)^{-1 / 2}$
$\vec{B}=c \vec{B}$
$\vec{B}$ magnetic field vector created at point $P$ by a moving charged particle $\mu_{0}$ magnetic permeability, $\mu_{0} / 4 \pi=10^{-7} \mathrm{Tsm} / \mathrm{C}$
$\vec{\nabla} \cdot \vec{E} \equiv \frac{\partial E_{x}}{\partial x}+\frac{\partial E_{y}}{\partial y}+\frac{\partial E_{z}}{\partial z}$
$\vec{\nabla} \cdot \vec{E}=\frac{\rho}{\epsilon_{0}}$
$\vec{\nabla} \cdot \vec{E}$ divergence of the electric field
$\vec{\nabla}=\left[\begin{array}{lll}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\end{array}\right]$
$\vec{E}$ Electric field
$\vec{\nabla} \cdot \vec{E}$ divergence of the electric field
$\vec{\nabla}=\left[\begin{array}{lll}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\end{array}\right]$
$\vec{E}$ Electric field
$\rho$ charge density
$\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}$

## E13

$$
\begin{aligned}
& \vec{\nabla} \cdot \vec{E}= \begin{cases}\frac{\partial E_{z}}{\partial z} & \text { for a unidirectional field } \\
\frac{1}{s} \frac{d}{d s}\left(s E_{s}\right) & \text { for an axial field } \\
\frac{1}{r^{2}} \frac{d}{d r}\left(r^{2} E_{r}\right) & \text { for an radial field }\end{cases} \\
& \vec{\nabla} \cdot \vec{E} \text { divergence of the electric field } \\
& E_{s} \text { Electric field component away from the central axis } \\
& s \text { distance from central axis } \\
& E_{r} \text { Electric field component away from the center point } \\
& r \text { distance from center point } \\
& \vec{\nabla} \times \vec{B} \equiv\left[\begin{array}{l}
\frac{\partial}{\partial y} B_{z}-\frac{\partial}{\partial z} B_{y} \\
\frac{\partial}{\partial z} B_{x}-\frac{\partial}{\partial x} B_{z} \\
\frac{\partial}{\partial x} B_{y}-\frac{\partial}{\partial y} B_{x}
\end{array}\right] \\
& \vec{\nabla} \times \vec{B}=\mu_{0} \vec{J} \\
& \begin{aligned}
\vec{\nabla} \times \vec{B}=\left[\begin{array}{c}
(\vec{\nabla} \times \vec{B})_{x} \\
(\vec{\nabla} \times \vec{B})_{y} \\
(\vec{\nabla} \times \vec{B})_{z}
\end{array}\right]=\left[\begin{array}{c}
0 \\
\frac{\partial B_{x}}{\partial z} \\
\frac{-\partial B_{x}}{\partial y}
\end{array}\right]
\end{aligned} \text { or } \begin{array}{c}
{\left[\begin{array}{c}
\frac{-\partial B_{y}}{\partial z} \\
0 \\
\frac{\partial B_{y}}{\partial x}
\end{array}\right]}
\end{array} \text { or } \begin{array}{c}
{\left[\begin{array}{c}
\frac{\partial B_{z}}{\partial y} \\
\frac{-\partial B_{z}}{\partial x} \\
0
\end{array}\right] \text { for a unidirectional field }} \\
\\
\text { if } \vec{B}=B_{x} \hat{x}
\end{array} \quad \text { if } \vec{B}=B_{y} \hat{y} \quad \begin{array}{l}
\text { if } \vec{B}=B_{z} \hat{z}
\end{array} \\
& \vec{\nabla} \times \vec{B}=\left[\begin{array}{c}
(\vec{\nabla} \times \vec{B})_{u} \\
(\vec{\nabla} \times \vec{B})_{\phi} \\
(\vec{\nabla} \times \vec{B})_{z}
\end{array}\right]=\left[\begin{array}{c}
\frac{-\partial B_{\phi}}{\partial z} \\
0 \\
\frac{1}{u} \frac{\partial}{\partial u}\left(u B_{\phi}\right)
\end{array}\right] \text { for a circular field }
\end{aligned}
$$

|  | $\vec{E}$ Electric field |
| :--- | :--- |
| $\oint \vec{E} \cdot d \vec{A}=\frac{q_{\mathrm{enc}}}{\epsilon_{0}}$ |  |
|  | $q_{\mathrm{enc}}$ net charge vector |
|  | $\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}$ |
| $\oint \vec{B} \cdot d \vec{S}=\mu_{0} \dot{i}_{\mathrm{enc}}$ | $\vec{B}$ Magnetic field |
|  | $d \vec{S}$ tile vector |
|  | $i_{\text {enc }}$ signed current that flows through gaussian surface |
| $\mu_{0}$ magnetic permeability, $\mu_{0} / 4 \pi=10^{-7}$ Tsm $/ \mathrm{C}$ |  |

## E15

## Maxwell's Equations

| $\vec{\nabla} \cdot \vec{E}=\frac{\rho}{\epsilon_{0}}$ | $\vec{\nabla} \cdot \vec{E}=\frac{\rho}{\epsilon_{0}}$ |
| :--- | :--- |
| $\vec{\nabla} \times \vec{B}-\epsilon_{0} \mu_{0} \frac{\partial \vec{E}}{\partial t}=\mu_{0} \vec{J}$ | $\vec{\nabla} \times \vec{B}-\frac{1}{c} \frac{\partial \vec{E}}{\partial t}=\frac{1}{\epsilon_{0}}\left(\frac{\vec{J}}{c}\right)$ |
| $\vec{\nabla} \cdot \vec{B}=0$ | $\vec{\nabla} \cdot \vec{B}=0$ |
| $\vec{\nabla} \times \vec{E}+\frac{\partial \vec{B}}{\partial t}=0$ | $\vec{\nabla} \times \vec{E}+\frac{1}{c} \frac{\partial \vec{B}}{\partial t}=0$ |
| $\vec{\nabla} \cdot \vec{E}$ divergence of the electric field |  |
| $\vec{\nabla} \times \vec{B}$ curl of the magnetic field | $\vec{\nabla} \cdot \vec{B}$ divergence of the magnetic field |
| $\vec{\nabla}=\left[\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z}\right]$ | $\vec{\nabla} \times \vec{E}$ curl of the electric field |
| $\vec{B}$ Magnetic field |  |
| $\rho$ charge density | $\vec{B}=c \vec{B}$ |
| $c=3.00 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$ | $\vec{J}$ current density |
| $\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}$ | $\mu_{0}$ magnetic permeability, $\mu_{0} / 4 \pi=10^{-7}$ Tsm/C |

## E16

$$
\begin{aligned}
& E_{\phi}= \begin{cases}\frac{-1}{2} \frac{\partial B_{z}}{\partial t} u & \text { for } u<R \\
\frac{-1}{2} \frac{\partial B_{z}}{\partial t} \frac{R^{2}}{u} & \text { for } u>R\end{cases} \\
& \mathcal{E}=-\frac{d \Phi_{B}}{d t}
\end{aligned}
$$

$u$ distance from the solenoids axis
$B_{z}$ Magnetic field along axis direction $R$ radius of solenoid
$E_{\phi}$ Electric field in the counterclockwise $\phi$ direction
$\mathcal{E}=\oint \vec{E} \cdot d \vec{r}$ emf induced in a closed loop
$\Phi_{B}=\int \vec{B} \cdot d \vec{A}$ magnetic flux

$$
Q(t)=Q_{0} \cos (\omega t)
$$

$$
\mathcal{E}_{\mathrm{src}}(t)=\mathcal{E}_{0} \sin (\omega t)
$$

$$
\mathcal{E}_{2}=\left(\frac{N_{2}}{N_{1}}\right) \mathcal{E}_{1}
$$

$$
i_{1}(t)=\frac{\mathcal{E}_{0}}{L \omega} \sin \left(\omega t-\frac{1}{2} \pi\right)
$$

$$
\begin{aligned}
& |\mathcal{E}|=L\left|\frac{d I}{d t}\right| \\
& L=\mu_{0} \pi r^{2} \frac{N^{2}}{\ell} \\
& I(t)=I_{0} e^{-(R / L) t} \\
& U^{\text {th }}=\frac{1}{2} L I_{0}^{2} \\
& U_{0}^{B}=\frac{\pi r^{2} \ell}{2 \mu_{0}}\left|\vec{B}_{0}\right|^{2} \\
& u_{E M}=\frac{1}{2}\left(\epsilon_{0}|\vec{E}|^{2}+\frac{|\vec{B}|^{2}}{\mu_{0}}\right) \\
& =\frac{\epsilon_{0}}{2}\left(|\vec{E}|^{2}+|\vec{B}|^{2}\right) \\
& i=\frac{-d Q}{d t}
\end{aligned}
$$

$\mathcal{E}=\oint \vec{E} \cdot d \vec{r}$ self-induced emf in a coil or loop
$L$ inductance
$I$ current flowing through the loop
$L$ inductance of a long solenoid
$\mu_{0}$ magnetic permeability, $\mu_{0} / 4 \pi=10^{-7} \mathrm{Tsm} / \mathrm{C}$
$r$ radius of solenoid
$N$ number of turns
$\ell$ length of solenoid
$I$ current in an inductor whose $\mathcal{E}$ pushes this current through a resister
$I_{0}=I(0)$
$R$ resistance of the resistance
$L$ inductance of the inductor
$U^{\text {th }}$ Energy dissipated by a resistor in an LR circuit $L$ inductance
$I_{0}$ initial current
$U_{0}^{B}$ magnetic field's energy inside a long solenoid at $t=0$
$r$ radius of solenoid, $\ell$ length of solenoid
$\vec{B}_{0}$ initial magnetic field
$u_{E M}$ energy density of an electromagnetic field
$\epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{~N} \cdot \mathrm{~m}^{2}}$
$\vec{E}$ Electric field
$\vec{B}$ Magnetic Field
$\vec{B}=c \vec{B}$
$i$ current in an LC circuit
$Q$ positive charge on one of the capacitor's plates
$Q$ positive charge on one of the capacitor's plates
$\omega=1 / \sqrt{L C}$
$L$ inductance, $C$ capacitance
$\mathcal{E}_{\text {src }}$ emf that the source imposes on the primary coil
$\mathcal{E}_{1}, \mathcal{E}_{2}$ emf in coil 1,2
$N_{1}, N_{2}$ number of turns in coil 1,2
$i_{1}$ current flowing in the primary coil

|  | $c$ speed of light |
| :---: | :---: |
| avg EM wave intensity $=c \epsilon_{0}\left[\|\vec{E}\|^{2}\right]$ avg | $\left[\|\vec{E}\|^{2}\right]_{\text {avg }}$ average of the electric field's squared magnitude $\Delta U_{\mathrm{EM}} / \Delta t$ Average power per unit time |
| $=\frac{1}{A} \frac{\Delta U_{\mathrm{EM}}}{\Delta t}=c\left[u_{\mathrm{EM}}\right]_{\mathrm{avg}}$ | $A$ area that the EM wave deposits energy onto $u_{\text {EM }}$ energy density of EM wave |
| $u_{\mathrm{EM}}=\epsilon_{0}\|\vec{E}\|^{2}$ | $u_{\text {EM }}$ energy density of EM wave $\vec{E}$ Electric field |
|  | $\vec{E}_{w}, \vec{B}_{w}$ Electric, Magnetic field of the EM wave at at a point $P$, caused by an accelerating particle <br> $q$ charge of particle |
| $\left\|\vec{E}_{w}\right\|=c\left\|\vec{B}_{w}\right\|=\frac{1}{4 \pi \epsilon_{0}} \frac{\|q \\| \vec{a}\| \sin \theta}{c^{2} R}$ | $\vec{a}$ acceleration of particle |
|  | $R$ distance from the accelerating particle to $P$ $\theta$ angle between $\vec{a}$ and $\vec{P}$ |
|  | $U_{\text {EM }}$ Energy contained in an EM wave $q$ charge of particle |
|  | $\vec{a}$ acceleration of particle |
| $U_{\mathrm{EM}}=\frac{\|q\|^{2}\|\vec{a}\|^{2}\left[\sin ^{2} \theta\right]_{\mathrm{avg}} \tau}{4 \pi \epsilon_{0} c^{3}}$ | $\tau$ brief time |
|  | $R$ distance from the accelerating particle to $P$ |
|  | $\theta$ angle between $\vec{a}$ and $\vec{P}$ $c$ speed of light |
|  | $P$ power that a particle radiates in the form of an EM wave |
| $P=\frac{q^{2}\|\vec{a}\|^{2}}{6 \pi \epsilon_{0} c^{3}}$ | $q$ charge of particle |
|  | $\vec{a}$ acceleration of particle |
|  | $c$ speed of light |

## Some Physical Constants

| Speed of light | c | $3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| :---: | :---: | :---: |
| Gravitational constant | G | $6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}$ |
| Coulomb's constant | $1 / 4 \pi \varepsilon_{0}$ | $8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$ |
| Permittivity constant | $\varepsilon_{0}$ | $8.85 \times 10^{-12} \mathrm{C}^{2} /\left(\mathrm{N} \cdot \mathrm{m}^{2}\right)$ |
| Permeability constant | $\mu_{0}$ | $4 \pi \times 10^{-7} \mathrm{~N} / \mathrm{A}^{2}$ |
| Planck's constant | $h$ | $6.63 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ |
| Boltzmann's constant | $k_{B}$ | $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ |
| Elementary charge | $e$ | $1.602 \times 10^{-19} \mathrm{C}$ |
| Electron mass | $m_{e}$ | $9.11 \times 10^{-31} \mathrm{~kg}$ |
| Proton mass | $m_{p}$ | $1.673 \times 10^{-27} \mathrm{~kg}$ |
| Neutron mass | $m_{n}$ | $1.675 \times 10^{-27} \mathrm{~kg}$ |
| Avogadro's number | $N_{A}$ | $6.02 \times 10^{23}$ |
| Commonly Used Physical Data |  |  |
| $\text { Gravitational field strength } g=\|\vec{g}\|$(near the earth's surface) |  | $9.80 \mathrm{~N} / \mathrm{kg}=9.80 \mathrm{~m} / \mathrm{s}^{2}$ |
| Mass of the earth $M_{e}$ |  | $5.98 \times 10^{24} \mathrm{~kg}$ |
| Radius of the earth $R_{e}$ |  | 6380 km (equatorial) |
| Mass of the sun $M_{\odot}$ |  | $1.99 \times 10^{30} \mathrm{~kg}$ |
| Radius of the sun $R_{\odot}$ |  | $696,000 \mathrm{~km}$ |
| Mass of the moon |  | $7.36 \times 10^{22} \mathrm{~kg}$ |
| Radius of the moon |  | 1740 km |
| Distance to the moon |  | $3.84 \times 10^{8} \mathrm{~m}$ |
| Distance to the sun |  | $1.50 \times 10^{11} \mathrm{~m}$ |
| Density of water ${ }^{+}$ |  | $1000 \mathrm{~kg} / \mathrm{m}^{3}=1 \mathrm{~g} / \mathrm{cm}^{3}$ |
| Density of air ${ }^{+}$ |  | $1.2 \mathrm{~kg} / \mathrm{m}^{3}$ |
| Absolute zero |  | $0 \mathrm{~K}=-273.15^{\circ} \mathrm{C}=-459.67^{\circ} \mathrm{F}$ |
| Freezing point of water ${ }^{\ddagger}$ |  | $273.15 \mathrm{~K}=0^{\circ} \mathrm{C}=32^{\circ} \mathrm{F}$ |
| Boiling point of water ${ }^{\ddagger}$ |  | $373.15 \mathrm{~K}=100^{\circ} \mathrm{C}=212^{\circ} \mathrm{F}$ |
| Normal atmospheric pressure |  | 101.3 kPa |
| ${ }^{\dagger}$ At normal atmospheric pressure and $20^{\circ} \mathrm{C}$. <br> ${ }^{\ddagger}$ At normal atmospheric pressure. |  |  |

Standard Metric Prefixes
(for powers of 10)

| Power | Prefix | Symbol |
| :--- | :--- | :--- |
| $10^{18}$ | exa | E |
| $10^{15}$ | peta | P |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-2}$ | centi | C |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

## Useful Conversion Factors

```
1 meter = 1 m = 100 cm = 39.4 in = 3.28 ft
1 mile = 1 mi = 1609 m = 1.609 km = 5280 ft
1 inch == 1 in = 2.54 cm
1 light-year = 1 ly =9.46 Pm = 0.946 }\times1\mp@subsup{0}{}{16}\textrm{m
1 minute = 1 min = 60 s
1 hour = 1 h = 60 min = 3600 s
1 day = 1d = 24 h = 86.4 ks=86,400 s
1 year = 1 y = 365.25 d=31.6 Ms=3.16 \times 10}\mp@subsup{0}{}{7}\textrm{s
1 newton = 1 N = 1 kg.m}/\mp@subsup{\textrm{s}}{}{2}=0.225\textrm{lb
1 joule = 1 J = 1 N}\cdot\textrm{m}=1\textrm{kg}\cdot\mp@subsup{\textrm{m}}{}{2}/\mp@subsup{\textrm{s}}{}{2}=0.239\textrm{cal
1 watt = 1 W = 1 J/s
1 pascal = 1 Pa = 1 N/m}\mp@subsup{m}{}{2}=1.45\times1\mp@subsup{0}{}{-4}\textrm{psi
1 kelvin (temperature difference) = 1 K=1 钅 = 1.8 %
1 radian = 1 rad = 57.3 }=0.1592 rev
1 revolution = 1 rev = 2\pi rad = 360
1 cycle = 2\pi rad
1 hertz=1 Hz=1 cycle/s
```

$1 \mathrm{~m} / \mathrm{s}=2.24 \mathrm{mi} / \mathrm{h}=3.28 \mathrm{ft} / \mathrm{s}$
$1 \mathrm{mi} / \mathrm{h}=-1.61 \mathrm{~km} / \mathrm{h}=0.447 \mathrm{~m} / \mathrm{s}=1.47 \mathrm{ft} / \mathrm{s}$
1 liter $=1 \mathrm{l}=(10 \mathrm{~cm})^{3}=10^{-3} \mathrm{~m}^{3}=0.0353 \mathrm{ft}^{3}$
$1 \mathrm{ft}^{3}=1728 \mathrm{in}^{3}=0.0283 \mathrm{~m}^{3}$
1 gallon $=1 \mathrm{gal}=0.00379 \mathrm{~m}^{3}=3.79 \mathrm{l} \approx 3.8 \mathrm{~kg} \mathrm{H}_{2} \mathrm{O}$
Weight of 1-kg object near the earth $=9.8 \mathrm{~N}=2.2 \mathrm{lb}$
1 pound $=1 \mathrm{lb}=4.45 \mathrm{~N}$
1 calorie $=$ energy needed to raise the temperature of 1 g of $\mathrm{H}_{2} \mathrm{O}$ by $1 \mathrm{~K}=4.186 \mathrm{~J}$
1 horsepower $=1 \mathrm{hp}=746 \mathrm{~W}$
1 pound per square inch $=6895 \mathrm{~Pa}$
1 food calorie $=1 \mathrm{Cal}=1 \mathrm{kcal}=1000 \mathrm{cal}=4186 \mathrm{~J}$
1 electron volt $=1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$

$$
\begin{array}{ll}
T=\left(\frac{1 \mathrm{~K}}{1^{\circ} \mathrm{C}}\right)\left(T_{[\mathrm{C}]}+273.15^{\circ} \mathrm{C}\right) & T_{[\mathrm{C}]}=\left(\frac{5^{\circ} \mathrm{C}}{9^{\circ} \mathrm{F}}\right)\left(T_{[\mathrm{F}]}-32^{\circ} \mathrm{F}\right) \\
T=\left(\frac{5 \mathrm{~K}}{9^{\circ} \mathrm{F}}\right)\left(T_{[\mathrm{F}]}+459.67^{\circ} \mathrm{F}\right), & T_{[\mathrm{F}]}=32^{\circ} \mathrm{F}+\left(\frac{9^{\circ} \mathrm{F}}{5^{\circ} \mathrm{C}}\right) T_{[\mathrm{C}]}
\end{array}
$$

## Electromagnetic Units and Conversion Factors

1 coulomb $=1 \mathrm{C}=$ unit of charge $=$ total charge of $6.242 \times 10^{18}$ protons
$1 \mathrm{~N} / \mathrm{C}=$ unit of electric field strength $=1 \mathrm{~V} / \mathrm{m}$
1 volt $=1 \mathrm{~V}=$ unit of energy per unit charge $=1 \mathrm{~V}=1 \mathrm{~J} / \mathrm{C}=1 \mathrm{~N} \cdot \mathrm{~m} / \mathrm{C}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} /\left(\mathrm{C} \cdot \mathrm{s}^{2}\right)$
1 ampere $=1 \mathrm{~A}=$ unit of current $=1 \mathrm{C} / \mathrm{s}$
$1 \mathrm{ohm}=1 \Omega=$ unit of resistance $=1 \mathrm{~V} / \mathrm{A}=1 \mathrm{~J} \cdot \mathrm{~s} / \mathrm{C}^{2}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} /\left(\mathrm{C}^{2} \mathrm{~s}\right)$
1 farad $=1 \mathrm{~F}=$ unit of capacitance $=1 \mathrm{C} / \mathrm{V}=1 \mathrm{~s} / \Omega=1 \mathrm{C}^{2} / \mathrm{J}=\mathrm{C}^{2} \mathrm{~s}^{2} /\left(\mathrm{kg} \cdot \mathrm{m}^{2}\right)$
1 watt $=1 \mathrm{~W}=$ unit of power (rate of energy conversion) $=1 \mathrm{~J} / \mathrm{s}=1 \mathrm{~V} \cdot \mathrm{~A}=1 \mathrm{~V}^{2} / \Omega$
1 tesla $=1 \mathrm{~T}=$ unit of magnetic field strength $=1 \mathrm{~N} \cdot \mathrm{~s} /(\mathrm{C} \cdot \mathrm{m})=1 \mathrm{~kg} /(\mathrm{C} \cdot \mathrm{s})$
1 gauss $=1 \mathrm{G}=$ unit of magnetic field strength $=10^{-4} \mathrm{~T}$
1 henry $=1 \mathrm{H}=$ unit of inductance $=\mathrm{V} \cdot \mathrm{s} / \mathrm{A}=1 \Omega \cdot \mathrm{~s}=\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{C}^{2}$
units of conductivity $=(\Omega \cdot \mathrm{m})^{-1}$
units of current density $=A / \mathrm{m}^{2}$
$\vec{B}=c \vec{B}=$ represents the magnetic field expressed N/C (same units as electric field)
$\left(\mu_{0} \varepsilon_{0}\right)^{-1}=c^{2} \quad \Rightarrow \quad 1 /\left(4 \pi \varepsilon_{0}\right)=\mu_{0} c^{2} / 4 \pi$
$\left(c \varepsilon_{0}\right)^{-1}=377 \Omega$

## Some Useful Indefinite Integrals

$\int x^{n} d x=\frac{1}{n+1} x^{n+1} \quad(n \neq-1) \quad \int \frac{d x}{\left(x^{2}+a^{2}\right)^{1 / 2}}=\ln \left[x+\left(x^{2}+a^{2}\right)^{1 / 2}\right] \quad \int \frac{x d x}{\left(x^{2}+a^{2}\right)^{1 / 2}}=\left(x^{2}+a^{2}\right)^{1 / 2}$
$\begin{array}{lll}\int \frac{d x}{x}=\ln |x| & \int \frac{d x}{x^{2}+a^{2}}=\frac{1}{a} \tan ^{-1}\left(\frac{x}{a}\right) & \int \frac{x d x}{\left(x^{2}+a^{2}\right)^{3 / 2}}=\frac{-1}{\left(x^{2}+a^{2}\right)^{1 / 2}} \\ \int e^{a x} d x=\frac{1}{a} e^{a x} & \int \frac{d x}{\left(x^{2}+a^{2}\right)^{3 / 2}}=\frac{1}{a^{2}} \frac{x}{\left(x^{2}+a^{2}\right)^{1 / 2}} & \cdot \int \frac{x^{2} d x}{\left(x^{2}+a^{2}\right)^{3 / 2}}=\frac{-x}{\left(x^{2}+a^{2}\right)^{1 / 2}}+\ln \left[x+\left(x^{2}+a^{2}\right)^{1 / 2}\right]\end{array}$

## Standard Electric and Magnetic Field Patterns

| Field pattern $(\vec{F}=\vec{E} \text { or } \vec{B})$ | Created by | Divergence | Curl | Gaussian Surface (G'S) <br> Amperian Loop (AL) |
| :---: | :---: | :---: | :---: | :---: |
| General | (anything) | $\vec{\nabla} \cdot \vec{F}=\frac{\partial F_{x}}{\partial x}+\frac{\partial F_{y}}{\partial y}+\frac{\partial F_{z}}{\partial z}$ | $\vec{\nabla} \times \vec{F}=\left[\begin{array}{l}\frac{\partial F_{z}}{\partial y}-\frac{\partial F_{y}}{\partial z} \\ \frac{\partial F_{x}}{\partial z}-\frac{\partial F_{z}}{\partial x} \\ \frac{\partial F_{y}}{\partial x}-\frac{\partial F_{x}}{\partial y}\end{array}\right]$ | (not applicable) |
| Unidirectional $\left(\vec{F}=F_{z} \hat{z}\right)$ | infinite flat plate (both $\vec{E}$ and $\vec{B}$ ) infinite solenoid $(\vec{B})$ | $\vec{\nabla} \cdot \vec{F}=\frac{\partial F_{z}}{\partial z}$ | $\vec{\nabla} \times \vec{F}=\left[\begin{array}{c}\frac{\partial F_{z}}{\partial y} \\ -\frac{\partial F_{z}}{\partial x} \\ 0\end{array}\right]$ | GS: rectangular solid AL: rectangle |
| $\begin{aligned} & \text { Axial } \\ & \left(\vec{F}=F_{s} \hat{s}\right) \end{aligned}$ | infinite cylindrically symmetric object ( $\vec{E}$ ) | $\vec{\nabla} \cdot \vec{F}=\frac{1}{s} \frac{\partial}{\partial s}\left(s F_{s}\right)$ | $\vec{\nabla} \times \vec{F}=0$ (identically) ${ }^{\text {c }}$ | GS: cylindrical can AL: (not applicable) |
| $\begin{aligned} & \text { Radial } \\ & (\vec{F}=F, \hat{r}) \end{aligned}$ | spherically symmetric object ( $\vec{E}$ ) | $\vec{\nabla} \cdot \vec{F}=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} F_{r}\right) .$ | $\vec{\nabla} \times \vec{F}=0 \text { (identically) }$ | GS: sphere <br> AL: (not applicáble) |
| Circular $\left(\vec{F}=F_{\phi} \hat{\phi}\right)$ | infinite cylindrically symmetric situation where current flows parallel to its axis $(\vec{B})$ or where $\partial \vec{B} / \partial t$ is parallel to the axis ( $\vec{E}$ ) | $\vec{\nabla} \cdot \vec{F}=0$ (identically) | $\vec{\nabla} \times \vec{F}=\left[\begin{array}{c}{[\vec{\nabla} \times \vec{F}]_{u}} \\ \vdots \vec{\nabla} \times \vec{F}]_{\phi} \\ \vdots \vec{i} \times \vec{F}]_{z}\end{array}\right]=\left[\begin{array}{c}-\frac{\partial F_{\phi}}{\partial z} \\ 0 \\ {\left[\begin{array}{l}\text { a }\end{array}\right.} \\ \frac{1}{u} \frac{\partial}{\partial u}\left(u F_{\phi}\right)\end{array}\right]$ | GS: (not applicable) <br> AL: circle |

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