# Section 1.4: Continuity

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# Definition of Continuity

**Definition (Continuity)**

A function $f(x)$ is said to be **continuous** at a point $x = c$ if and only if $\lim_{x\to c} f(x) = f(c)$.

**Example (Continous Functions)**

1. $\lim_{x\to 2}\left(x^2+4\right) = 8$
2. $\lim_{z\to -4}(7z-12) = -40$
3. $\lim_{h\to 0}\left(\frac{3x-1}{7x-2}\right) = \frac{1}{2}$

**Example**

Using the graph of $f$, determine the values below. Use them to discuss the continuity of $f$.

1. $\lim_{x\to -2} f(x) =$
2. $f(-2) =$
3. Is $f(x)$ continuous at $x = -2$?
4. $\lim_{x\to 2^-} f(x) =$
5. $\lim_{x\to 2^+} f(x) =$
6. Is $f(x)$ continuous at $x = 2$?

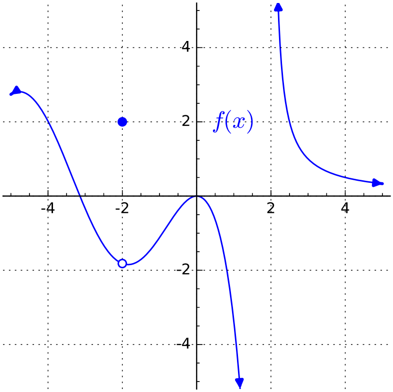


Figure: Graph of $f(x)$

# Types of Discontinuity

**Definition (Discontinuity)**

A function is **discontinuous** if and only if it is NOT continuous. The three types of discontinuities are

1. removable:

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \neq f(c),$$

2. infinite:

$$\lim_{x \to c^-} f(x) = \pm\infty \text{ or } \lim_{x \to c^+} f(x) \pm \infty,$$

3. jump:

$$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x).$$

Let $g(x)$ be the function graphed below.

The function $g(x)$ has the following types of discontinuities at the specified value of $x$. For clarification, there is not a discontinuity $x = -5$.

1. Removable discontinuity at $x = -5$
2. Infinite discontinuity at $x = -1$
3. Jump discontinuity at $x = 4$

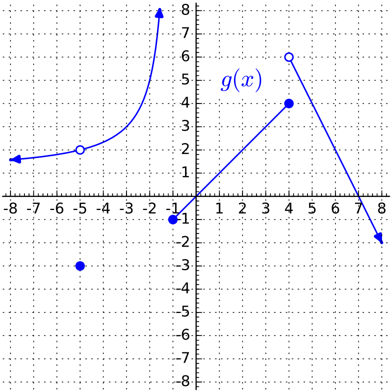


Figure: Graph of $g(x)$

# Types of Discontinuities – You Try

The function $h(x)$ is graphed below. Determine where $h(x)$ is discontinuous and specify the discontinuity.

| $x$-value | | | |
|---|---|---|---|
| discontinuity | | | |

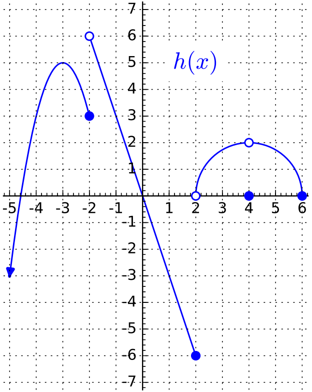


Figure: Graph of $h(x)$

# Left- and right-continuity

**Definition (Left- and right-continuity)**

1. A function is said to be **left-continuous** if
$$\lim_{x \to c^-} f(x) = f(c).$$
2. A function is said to be **right-continuous** if
$$\lim_{x \to c^+} f(x) = f(c).$$

# Left- and right-continuity – Example

Let $g(x)$ be the function graphed below.

The following are true about $g(x)$.

1. $g(x)$ is neither left- nor right-continuous at $x = -5$.
2. $g(x)$ is right-continuous at $x = -1$.
3. $g(x)$ is left-continuous at $x = 4$.

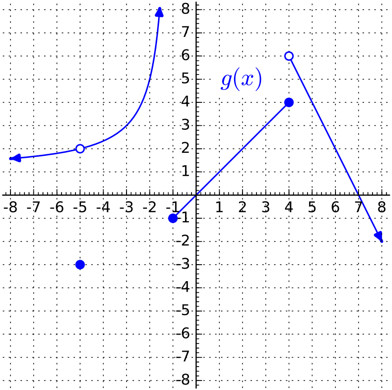


Figure: Graph of $g(x)$