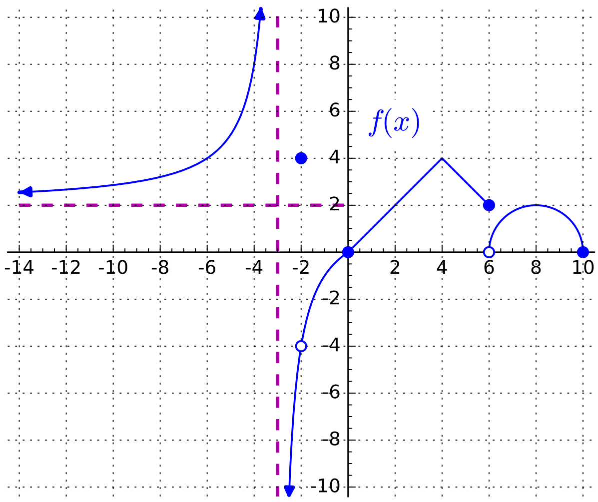
All work on this assessment should be your own. The technology allowed on this test includes: Desmos (`desmos.com/calculator`) and a TI-84 calculator (or less). This exam has 10 questions, for a total of 100 points.

1. (18 points) The graph of $f(x)$ is shown below. Answer each of the following questions concerning its behavior. **Infinite limits should be expressed using $-\infty$ or ∞.** Otherwise, write DNE.



(a) $\lim_{x\to-\infty} f(x) =$ ______

(b) $\lim_{x\to-3^-} f(x) =$ ______

(c) $\lim_{x\to-3^+} f(x) =$ ______

(d) $\lim_{x\to-2^-} f(x) =$ ______

(e) $\lim_{x\to-2^+} f(x) =$ ______

(f) $\lim_{x\to-2} f(x) =$ ______

(g) $\lim_{x\to4} f(x) =$ ______

(h) $\lim_{x\to6^-} f(x) =$ ______

(i) $\lim_{x\to6^+} f(x) =$ ______

(j) $f(-2) =$ ______

(k) $f(6) =$ ______

(l) $\lim_{x\to10^+} f(x) =$ ______

(m) Circle the letter to indicate which response best completes the following sentences.

i. The function f has _________-continuity at $x = -2$.

A. left B. right C. left- and right D. none of these

ii. The function f has _________-continuity at $x = 0$.

A. left B. right C. left- and right D. none of these

iii. The function f has _________-continuity at $x = 6$.

A. left B. right C. left- and right D. none of these

(n) Fill in the table with the x-values on which this function is discontinuous, then identify what type of discontinuity exists at the given x-value.

x-value			
Discontinuity			

2. (16 points) In the following section, box TRUE or FALSE. Example: $\boxed{\text{TRUE}}$ or $\boxed{\text{FALSE}}$.

(a) TRUE / FALSE

If $\lim_{x\to 3^+} f(x) = \infty$, then the graph of f has a horizontal asymptote at $x = 3$.

(b) TRUE / FALSE

If $\lim_{x\to c} f(x) = 10$, then $f(c) = 10$.

(c) TRUE / FALSE

If $0 < |x - c| < \delta$, then $x \in (c - \delta, c) \cup (c, c + \delta)$.

(d) TRUE / FALSE

If $f(3) = -5$ and $f(9) = -2$, then there must be a value c at which $f(c) = -3$.

(e) TRUE / FALSE

$\forall_{c\in(0,\,5)}\ f$ is continuous $\Rightarrow \lim_{x\to c} f(x)$ exists.

(f) TRUE / FALSE

The function $f(x) = \sec(x)$ is continuous at $x = \dfrac{\pi}{2}$.

(g) TRUE / FALSE

If $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to\infty} (f(x) - g(x)) = 0$.

(h) TRUE / FALSE

If a limit has an indeterminate form, then the limit does not exist.

3. (6 points) Consider $\lim_{x \to 1} \left(x^2 - 3\right) = -2$. Define the meaning of this limit using the ***absolute value*** definition of a limit.

4. (6 points) Consider $\lim_{x \to 2} x^3 = 8$. Use graphs and/or algebra to approximate the largest value of δ such if $x \in (c - \delta, c) \cup (c, c + \delta)$, then $f(x) \in (L - \varepsilon, L + \varepsilon)$, where $\varepsilon = 0.2$.

5. (10 points) Use an epsilon-delta proof to prove that $\lim_{x \to 3} (5x - 7) = 8$.

6. (24 points) Use algebra, limit rules, or the Squeeze Theorem to calculate the following six limits. If you elect to use the Squeeze Theorem, then you must provide me with an $\ell(x)$ and a $u(x)$ such that $\ell(x) \leq f(x) \leq u(x)$. **You may use graphs to verify your answer, but that alone cannot be your sole reason for your answer**. Each correct response is worth 4 points.

   (a) $\lim_{x\to 3}\left(\dfrac{x^2-9}{x-3}\right)$

   (b) $\lim_{x\to 0}\left(\dfrac{4e^x-4}{e^{2x}+3e^x-4}\right)$

   (c) $\lim_{h\to 0}\left(\dfrac{\frac{1}{-4+h}+\frac{1}{4}}{h}\right)$

   (d) $\lim_{x\to 0}\left(1+\dfrac{x}{4}\right)^{8/x}$

   (e) $\lim_{x\to 0}\left(\dfrac{\sin(6x)}{7x}\right)$

   (f) $\lim_{x\to\infty}\left(\ln(9x^{88})-\ln(3x^{88})\right)$

7. (20 points) Each question is worth 5 points. There is no partial credit for this section. Write your choice on the line provided.

   (a) $\lim_{x\to\infty} \dfrac{2x^2+1}{(2-x)(2+x)}$ is

   A. -4. B. -2. C. 1. D. 2. E. nonexistent.

   (a) __________

   (b) $\lim_{x\to 0} \dfrac{|x|}{x}$ is

   A. -1. B. 0. C. 1. D. nonexistent. E. none of these.

   (b) __________

   (c) Let $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 4, & \text{if } x = 1 \end{cases}$. Which of the following are true?

   I. $\lim_{x\to 1} f(x)$ exists. II. $f(1)$ exists. III. f is continuous at $x = 1$.

   A. I only B. II only C. I and II only D. all of them E. none of them

   (c) __________

   (d) $\lim_{x\to\infty} \sin(x)$ is

   A. -1. B. 0. C. 1. D. ∞. E. nonexistant.

   (d) __________

   (e) $\lim_{x\to\infty} x \sin\left(\dfrac{1}{x}\right)$ is

   A. -1. B. 0. C. 1. D. ∞. E. nonexistent.

   (e) __________

Challenge Questions

You may choose to do ***TWO*** *of the following Challenge Problems for extra credit. These questions are wroth two points each beyond the total points of the exam. They will receive no partial credit. You must have the correct response and defense of your answer or no credit will be awarded.*

8. For each given function f, find a real number a that makes f continuous at $x = 0$, if possible. If not possible, explain why. You must have clearly written work to illustrate how you arrived at your conclusion.

   (a) (2 points (bonus)) $f(x) = \begin{cases} 3x+1, & x < 0 \\ 2x+a, & x \geq 0 \end{cases}$

   (b) (2 points (bonus)) $f(x) = \begin{cases} \frac{a}{x+2}, & x < 0 \\ 3, & x = 0 \\ ax+a, & x > 0 \end{cases}$

9. (2 points (bonus)) Use algebra, limit rules, or the Squeeze Theorem to calculate the following limit. If you elect to use the Squeeze Theorem, you must provide me with an $\ell(x)$ and a $u(x)$. **You may use graphs to verify your answer, but that alone cannot be your sole reason for your answer**

$$\lim_{x\to\infty} \frac{4x^2 + 2\cos(6x)}{x^2+1}$$

10. (2 points (bonus)) Use Desmos to determine the ***exact*** value of the following limit.

$$\lim_{n\to\infty} \sum_{i=0}^{n} \frac{(-1)^i}{2i+1}$$