LS30A - Lab 9

Aaron

Review

6. [Garfinkel F17 Final Exam] With the rise of multi-drug resistant bacterial infections over the last decade, researchers have begun to develop new ways to treat infections for the future post-antibiotic era, including the use of predatory and competitive bacteria.
   a. (4 pts) One of the predatory bacteria being studied is *Bdellovibrio bacteriovorus* (B), which feeds on other gut microbiota such as Lactobillus (L). Suppose that interaction is described by the following model:

$$L' = 0.3L(1 - \tfrac{L}{30}) - 0.05LB$$

$$B' = 0.01LB - 0.1B\ .$$

   Find and plot the nullclines and equilibria for this system. Be sure to visually distinguish the nullclines (color, dotted lines, etc.) and to give the coordinates of the equilibria.

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L-nullcline: $L' = 0.3L(1 - \frac{L}{30}) - 0.05LB = 0$

$$L[0.3 - \frac{L}{100} - 0.05B] = 0$$

$L = 0$ and $0.3 - \frac{L}{100} - 0.05B = 0$

$$0.05B = 0.3 - \frac{L}{100}$$

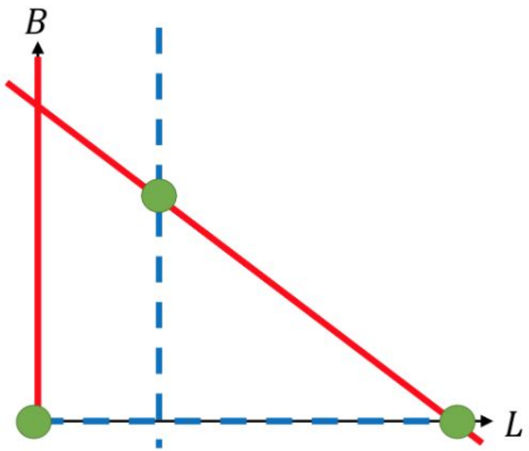
$$B = 6 - \frac{L}{5}$$

B-nullcline: $B' = 0.01LB - 0.1B = 0$

$$B[0.01L - 0.1] = 0$$

$B = 0$ and $0.01L = 0.1$

$$L = 10$$



Equilibria:

$(L, B) = (30, 0)$

$(L, B) = (10, 4)$

$(L, B) = (0, 0)$

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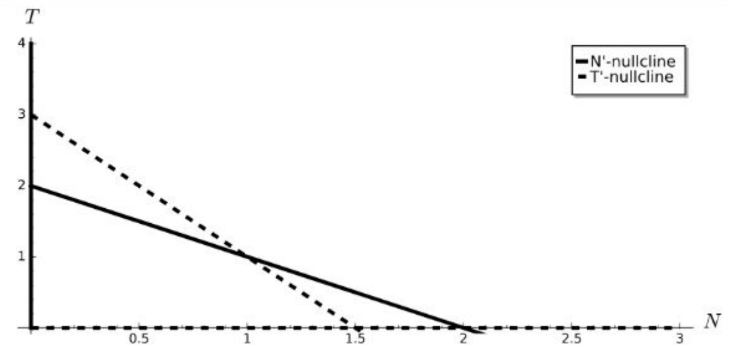
b. (6 pts) Two strains of bacteria *Bacteroides fragilis*, one non-toxigenic (N) and one toxigenic (T), compete for resources in the large intestine . Suppose that interaction is described by the following model:

$$N' = 4N - 2TN - 2N^2$$
$$T' = 3T - 2TN - T^2 .$$

Below is a graph of nullclines for that model. Use the test point method to sketch the vector field, determine the types of each equilibrium point, and determine if the two strains can coexist. Be sure to specify which test points you choose.



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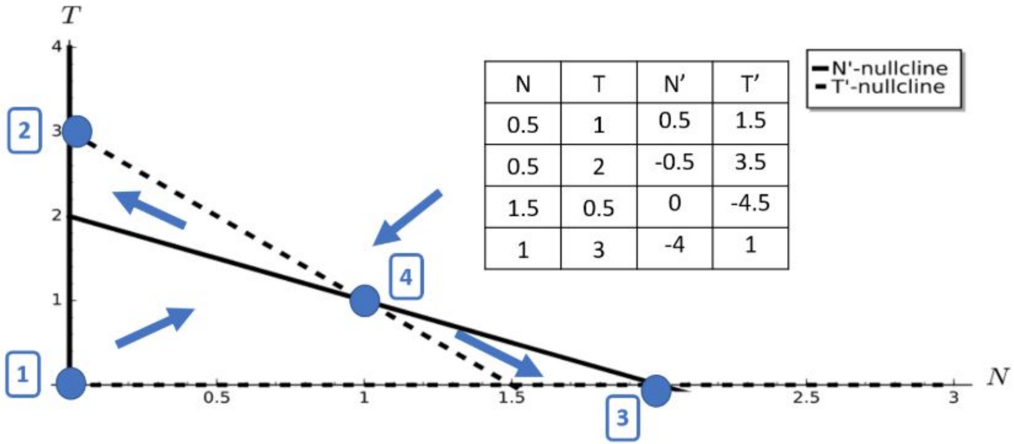

N	T	N′	T′
0.5	1	0.5	1.5
0.5	2	-0.5	3.5
1.5	0.5	0	-4.5
1	3	-4	1

#	N	T	Type of EP	
1	0	0	Unstable node	The two strains cannot coexist because the only equilibrium points at which both N and T are nonzero is unstable. Either N persists or T persists, not both.
2	0	3	Stable node	
3	2	0	Stable node	
4	1	1	Saddle point	

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7. We can model the concentration of lactose in the cell, as controlled by the lac operon, as $X' = \frac{X^2}{1+X^2} - rX$ (note: this is a more simplified version than in the textbook). For a model like this with a distinct inflow and outflow, we can easily see the equilibria and how changes in parameters like r affect the model's behavior if we graph the inflow and outflow separately vs. X. Equilibria occur where $X' = 0$ and thus where the inflow and outflow are equal. Note: the graph of $\frac{X^2}{1+X^2}$ is S-shaped starting at the origin and leveling off at 1 as shown below.

   a. Graph the inflow and outflow separately for $X' = \frac{X^2}{1+X^2} - rX$, where $r = 0.6$. Make sure you use different colors for inflow and outflow. Label the equilibria. For each equilibrium, is it stable, unstable, or semi-stable?

   b. Graph the inflow and outflow separately for $X' = \frac{X^2}{1+X^2} - rX$, where $r = 0.3$. Make sure you use different colors for inflow and outflow. Label the equilibria. For each equilibrium, is it stable, unstable, or semi-stable?

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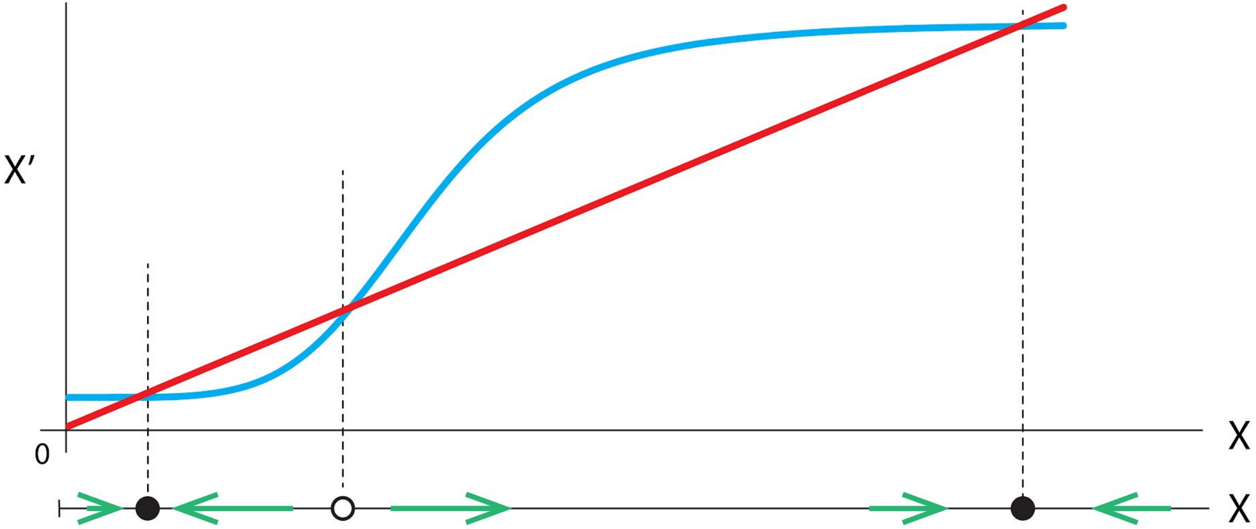

Figure 3.34: Rates of lactose importation (blue) and metabolic degradation (red) as functions of lactose concentration.

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a. **A bifurcation is a qualitative change in a system's behavior (such as a change in the number or type of equilibria) due to a gradual change in a parameter. Do you see a bifurcation in the graphs we drew? In this lab, you'll search for the parameter value at which this bifurcation takes place!**

When r=0.6,we have 1 equilibrium. When r=0.3, we have 3 equilibria. Since the number of parameters changed as we changed the parameter r,we must have a bifurcation between r=0.3 and r=0.6!