LS30A - Lab 8

Aaron

Review

- **Microsoft founder Paul Allen paid for a detailed census taking three years to estimate the population size of African elephants. This census found 352,271 elephants in 2016 in all of Africa and that the population was declining by about 8% per year.**
    - **Write a differential equation that reflects the change in this population.**
    - **What do we call this type of growth/decay?**
    - **What is the general differential equation and its solution for this type of growth/decay?**
    - **How many elephants do we expect to live in Africa in 2026?**

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- **Microsoft founder Paul Allen paid for a [detailed census] taking three years to estimate the population size of African elephants. This census found 352,271 elephants in 2016 in all of Africa and that the population was declining by about 8% per year.**
  - **Write a differential equation that reflects the change in this population.**

    Let E = # of elephants
    $E' = -0.08E$

  - **What do we call this type of growth/decay?**

    Exponential decay (since constant < 0)

  - **What is the general differential equation and its solution for this type of growth/decay?**

    $X' = rx$
    $X(t) = X(0) * e^{rt}$

  - **How many elephants do we expect to live in Africa in 2026?**

Since that is 10 years after the census, $t = 10$.
Since there were 352,271 elephants in 2016, $E(t=0) = 352{,}271$
$E' = -0.08E$
$E(t) = 352271 * e^{-0.08t}$
$E(t=10) = 352271 * e^{-0.08*10} = 158{,}285$ elephants in 2026!
That's less than half of the population in 2016!

Review

1. **We want to determine the area under the curve of f(x)=10 - 2x^2 between x= 0 and x=2.**
    a. **What method and equation should we use to approximate this area using a given x?**

    b. **Approximate the area using delta_x=0.5.**

    c. **What method and equation should we use calculate the exact area under the curve?**

    d. **Calculate the exact area under the curve.**

Review

1. **We want to determine the area under the curve of f(x)=10 - 2x^2 between x= 0 and x=2.**
   a. **What method and equation should we use to approximate this area using a given x?**

      Riemann sum:

      $$X(t) \approx X(0) + \sum_{k=0}^{k=n} V(k \cdot \Delta t) \cdot \Delta t$$

   b. **Approximate the area using delta_x=0.5.**

      Area under curve of $f(x) = 10 - 2x^2$ between $x = 0$ and $x = 2$

      $$f(0) * 0.5 + f(0.5) * 0.5 + f(1) * 0.5 + f(1.5) * 0.5 = 10 * 0.5 + 9.5 * 0.5 + 8 * 0.5 + 5.5 * 9.5 = 16.5$$

   c. **What method and equation should we use calculate the exact area under the curve?**

      We can use the reverse of our derivative rules to calculate a definite integral
      Fundamental Theorem of Calculus:

      $$X(t) - X(0) = \int_0^t X' \cdot dt$$

   d. **Calculate the exact area under the curve.**

      Since $f(x) = 10 - 2x^2$, $F(x) = 10x - 2/3 * x^3 + C$
      Fundamental Theorem of Calculus $\Longrightarrow$

      $$F(2) - F(0) = \int_0^2 10 - 2x^2 = (10x - 2/3 * x^3 + C) \text{ for } x = 2 -$$

      $$x = 1 = (10 * 2 - 2/3 * 2^3 + C) - (10 * 0 - 2/3 * 0 + C) = (20 - 16/3 + C) - (C) = 44/3 = 14.7$$

Review about Labs

1. (10 points) Plot the function $f(x) = x^2$in red for values of x ranging between -10 and 10. Plot the function $g(x) = x^3$ in green for values of x ranging between -5 and 5. Overlay these plots. (Note: you are not required to define the functions.)

2. (10 points) You are studying populations of penguins and marine iguanas on a beach in the Galapagos. Over five years, the penguin population at your study site has been 62, 93, 75, 56 and 76. In the same years, the marine iguana population has been 34, 21, 15, 25 and 34. Plot the system's states in penguin-marine iguana space, making the points green and large.

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Review about Labs

1. (10 points) Write a function that prints "It's hot" if the temperature is greater than 85, "It's cold" if temperature is less than 65, and "Not bad" if the temperature is between 65 and 85, inclusive. Test the function with three different temperatures.

2. (10 points) The script below should iterate the function $f(x) = 3x$ five times with an initial value of 1, but has five errors. Correct the errors and explain what each line does in a comment. If the script works correctly, it should generate the output at the bottom of the script. In a new cell, convert the script into a function that takes the number of iterations as input and returns the list of values. Test the function for three different numbers of iterations.

```
mult3 = [1]
nums = srange(0, 5
for n in nums
    test = 3*mult3[i]
    append.mult3(test)
Mult3 # Desired output: [1, 3, 9, 27, 81, 243]
```

Review about Labs

4. (10 points) Write a script that calculates the factorial of an integer n (written $n!$). To calculate a factorial, we take the number, n, and multiply it by all of the integers between 1 and n, inclusive. For example, 2! is 2 x 1 = 2 and 3! = 3 x 2 x 1 = 6. Test the script with two different values of n.

3. (10 points) The simulation script below has five errors. Correct the errors and explain what each line does in a comment.

```
var("N,P")
t = srange(0,100,0.1
sol=desolve_odeint([0.5*N - 0.01*N*P, 0.5*0.01*N*P - 0.2P], ics=[50,75], dvars=[N,P], times=t1)
list_plot((t, sol[:,0])) + plot(zip(t,sol[:,1]), color="red"")
```