LS30A - Lab 7

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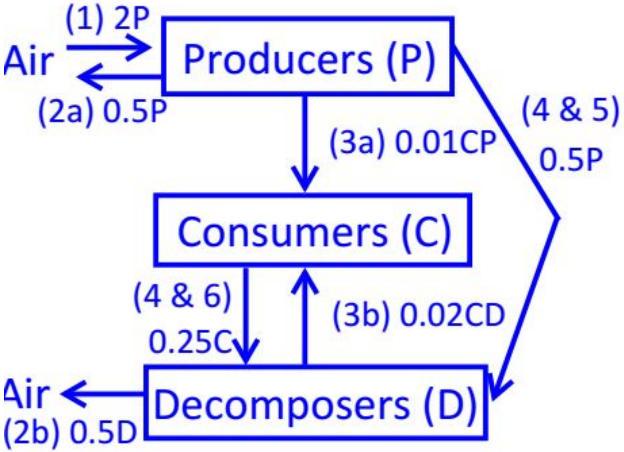
Review about Modelling

1. [Shevtsov Su14 Final] (10 pts) Any ecosystem can be divided into producers (P), which capture CO_2, consumers (C), which eat living organisms, and decomposers (D), which consume dead organisms. We can quantify the abundance of the different groups by the amount of carbon each one contains. Use the assumptions below to write a system of differential equations describing this ecosystem. (HINT: Start by drawing a diagram.)

- Producers capture carbon from the air at a rate proportional to their abundance with a proportionality constant of 2.
- Producers and decomposers lose carbon through respiration at a per-mass (like per-capita) rate of 0.5.
- The rate at which consumers eat producers is proportional to both their abundances with a proportionality constant of 0.01. The same applies to consumers eating decomposers, but with a proportionality constant of 0.02.
- All dead producers and consumers become food for decomposers.
- Producers lose carbon (due to death) at a per-mass rate of 0.5.
- Consumers lose carbon (due to death) at a per-mass rate of 0.25.

Review about Modelling

D increase 0.25C



Thus, the full system of differential equations is:

$$\boxed{\begin{cases} P' = 2P - 0.5P - 0.01CP - 0.5P \\ C' = 0.01CP + 0.02CD - 0.25C \\ D' = 0.5P + 0.25C - 0.02CD - 0.5D \end{cases}}$$

State variables:

P = abundance (amount of carbon) of producers

C = abundance (amount of carbon) of consumers

D = abundance (amount of carbon) of decomposers

P' = photosynthesis (1) - respiration (2a) - consumption (3a) - death (5)

$P' = 2P - 0.5P - 0.01CP - 0.5P$

C' = consumption of producers (3a) + consumption of decomposers (3b) - death (6)

$C' = 0.01CP + 0.02CD - 0.25C$

D' = decomposition of dead producers (4&5) + decomposition of dead consumers (4&6) - consumption (3b) - respiration (2b)

$D' = 0.5P + 0.25C - 0.02CD - 0.5D$

Review about Euler Method

1. [Shevstov Summer 2016 Midterm] A population of bacteria is growing at a per-capita rate of 2.5 bacteria per day.
    a. Write a differential equation modeling this system.

    b. Use Euler's method with a step size of 0.5 days and an initial population size of 1000 bacteria to estimate the population size after 1 day.

    c. How could you make the estimated value closer to the exact value?

Review about Euler Method

Use Euler's method with a step size of 0.5 days and an initial population size of 1000

X' = 2.5X

t	X_{old}	Δt	$X'(X_{old})$	$\Delta t * X'(X_{old})$	$X_{new} = X_{old} + \Delta t * X'(X_{old})$
0	1000	0.5	2500	1250	2250
0.5	2250	0.5	5625	2812.5	5062.5
1	5062.5				

We use Euler's method to estimate that the population after 1 day is 5,062.5 bacteria.

Review about derivatives

1. (F17 Midterm Garfinkel - 15 points) Use the function below to answer the following questions.

$$f(X)=3X^2+3X-5$$

   a. (4 points) Use the definition of a derivative to calculate the derivative of f(X) with respect to X.

   b. (2 points) Confirm your calculation above using derivative rules (be sure to state which rule(s) you used). See Appendix: Derivative Rules attached.

   c. (4 points) Find the equation of the linear approximation to f(X) at the point X=2.

   d. (5 points) Using this linear approximation, estimate the value of f at X=2.05.

Review about derivatives

a. (4 points) Use the definition of a derivative to calculate the derivative of f(X) with respect to X.

Derivative of $f(X)$ with respect to $X = f'(X) = lim_{\Delta X \to 0} \frac{f(X+\Delta X)-f(X)}{\Delta X}$

$= \frac{[3(X+\Delta X)^2+3(X+\Delta X)-5]-[3X^2+3X-5]}{\Delta X} = \frac{3[X^2+2X(\Delta X)+(\Delta X)^2]+3(X+\Delta X)-5-[3X^2+3X-5]}{\Delta X} = \frac{6X(\Delta X)+3(\Delta X)^2+3(\Delta X)}{\Delta X}$

$= 6X + 3\Delta X + 3$

Since $\Delta X \to 0$, $3\Delta X \to 0$, leaving us with $f'(X) = 6X + 3$

b. (2 points) Confirm your calculation above using derivative rules (be sure to state which rule(s) you used). See Appendix: Derivative Rules attached.

6X+3

c. (4 points) Find the equation of the linear approximation to f(X) at the point X=2.

Equations of linear approximations to the function $f(X)$ at the point $X = X_0$ are in the format: $\Delta f \approx \frac{df}{dX}|_{X=X_0} \Delta X$

$\frac{df}{dX} = f'(X) = 6X + 3$

$\frac{df}{dX}|_{X=2} = f'(2) = 6 * 2 + 3 = 15$

$\Delta f \approx 15\Delta X$

d. (5 points) Using this linear approximation, estimate the value of f at X=2.05. 13.75

Review about Lab7

1. **if statement (Don't forget the colon)**
   >, <, ==, !=, >=, <=, etc.

2. **if-else statement**
3. **if-elif-else statement**
4. **While Loops vs. For Loops**