All work on this lab should be the collective effort of all group members. Technology allowed on this lab includes: Desmos (`https://www.desmos.com/calculator`) and an approved TI calculator. This lab has 8 questions for a total of 60 points.

1. (20 points) Determine if the following statments are true or false. You must provide a justification for your answer.

   (a) $\boxed{\text{TRUE}}$ / FALSE

   If a number is divisible by 6, then it is divisible by 3.

   Solution: If a number, x, is divisible by 6, then it can be written of the from $x = 6y, y \in \mathbb{Z}$. This means that $x = 2 \cdot 3 \cdot y$, which is clearly divisible by 3.

   (b) TRUE / $\boxed{\text{FALSE}}$

   For all real numbers x and y, $\frac{x}{y} = 0$ if and only if $x = 0$.

   Solution: Statement is false because it says *for all* x and y and this is not possible, since if $y = 0$, then the value is undefined.

   (c) $\boxed{\text{TRUE}}$ / FALSE

   For all real numbers y there is a real number x such that $y = 2x + 4$.

   Solution: This is true since you can re-write the equation as $x = \dfrac{y-4}{2}$, which shows that for any value of y, x is going to also be a real number.

   (d) $\boxed{\text{TRUE}}$ / FALSE

   For all real numbers $x > 0$ and $y > 0$, if $x > y$, then $\dfrac{1}{x} < \dfrac{1}{y}$

   Solution: This is also true since if $x > 0$ and $y > 0$, if we have $x > y \to 1 > \frac{y}{x} \to \frac{1}{y} > \frac{1}{x}$.

2. Consider the following implication:

   If x is divisible by 12, then x is divisible by 3.

   (a) (2 points) Write the converse of the statement.

   Solution: If x is divisible by 3, then x is divisible by 12.

   The converse of the implication is TRUE / $\boxed{\text{FALSE}}$.

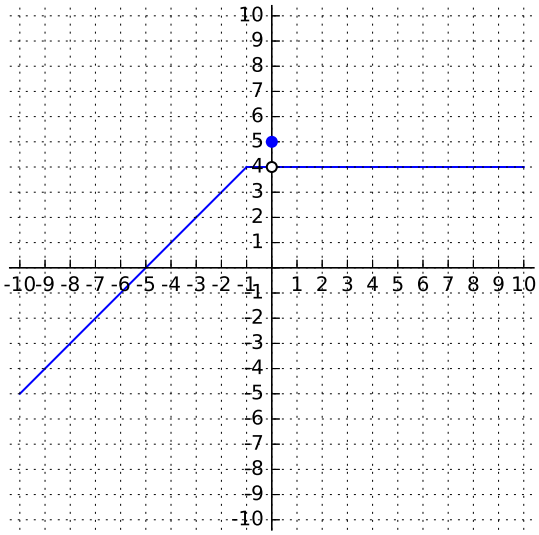
   (b) (2 points) Write the contrapositive of the statement.

   Solution: If x is NOT divisible by 3, then x is NOT divisible by 12.

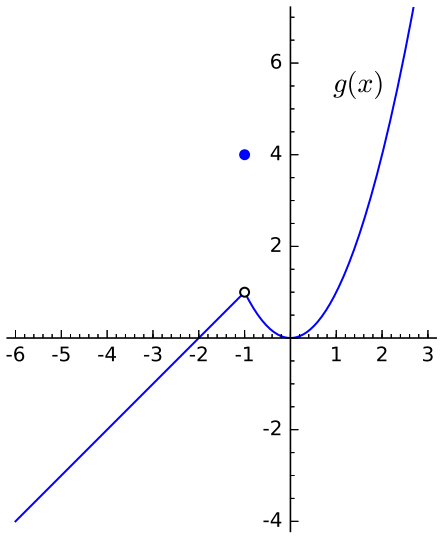
   The contrapositive of the implication is $\boxed{\text{TRUE}}$ / FALSE.

3. (6 points) Sketch the graph of a function that has the given limits and values. There is more than one correct answer.

- $\lim_{x \to -1} f(x) = 4$
- $\lim_{x \to \infty} f(x) = 4$
- $f(0) = 5$



4. (6 points) Let $g(x)$ be the function graphed below.



(a) $\lim_{x \to -1} g(x) =$ **1**

(b) $\lim_{x \to 1} g(x) =$ **1**

(c) $g(-1) =$ 4

(d) $\lim_{x \to 2} g(x) =$ 4

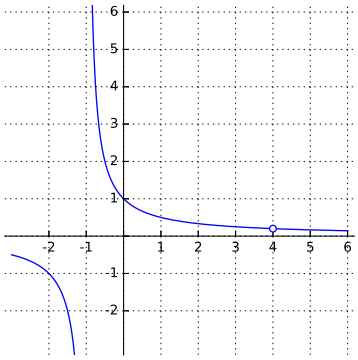
(e) $\lim_{x \to -\infty} g(x) = -\infty$

(f) $\lim_{x \to \infty} g(x) = \infty$

5. (5 points) Give an argument, be it a table of values, or a graph, that justifies your educated guess for the following limit:

$$\lim_{x\to 4} \frac{x-4}{(x+1)(x-4)}.$$

Solution: the graph of the function, shown below, indicates that a hole is created from the



cancellation in the rational function.

Therefore, if we look at the table of values, and values incrediably close to 4, then we have the following.

x	3.9	3.99	3.999	4	4.0001	4.01	4.01
$f(x)$	0.20408	0.204008	.020004001	undefined	0.199996	0.19996001	0.1996008

This indicates that the function is approaching 0.2, or $\frac{1}{5}$, as the x-values get closer and closer to $x = 4$.

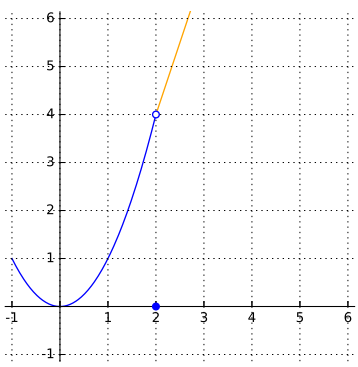
6. (4 points) Let $h(x)$ be the following piecewise-defined function:

$$h(x) = \begin{cases} 3x-2 & \text{if } x > 2 \\ 0 & \text{if } x = 2 \\ x^2 & \text{if } x < 2 \end{cases}.$$

Using the graph of the function, determine the value of $\lim_{x\to 2} h(x)$.

6. **4**

Solution: Below is the graph of $h(x)$.



7. (5 points) Write the following limit using the $\varepsilon - \delta$ definition of a limit.

$$\lim_{x \to 3} \left(x^2 - 4\right) = 5$$

Solution: For all $\varepsilon > 0$, there exists a $\delta > 0$, such that if $x \in (3 - \delta, 3) \cup (3, 3 + \delta)$, then $(x^2 - 4) \in (5 - \varepsilon, 5 + \varepsilon)$.

8. (10 points) Given that $\lim_{x \to 2} (2x + 1) = 5$, use algebra to approximate the largest value of δ such that

$$\text{if } x \in (2 - \delta, 2) \cup (2, 2 + \delta), \text{ then } f(x) \in (5 - \varepsilon, 5 + \varepsilon) \text{ where } \varepsilon = 0.01.$$

Solution: Since $\varepsilon = 0.01$, we have an epsilon-band that has been created with an upper y-value of 5.01 and a lower y-value of 4.99. We can determine the x-values that correspond to these values by solving the following linear equations:

$$\begin{aligned} 2x + 1 &= 5.01 \\ 2x &= 4.01 \\ x &= 2.005 \end{aligned} \qquad \begin{aligned} 2x + 1 &= 4.99 \\ 2x &= 3.99 \\ x &= 1.995 \end{aligned}$$

Since we are looking for the largest δ, this really means that we are look of rhte distance these x-values are around 2. A quick inspection of these values show that they are both 0.005 units from 2; therefore, we would choose $\delta = 0.005$.