E16M.4

Let the z direction lie in the direction of the current and define the u direction to be perpendicular from z, pointing away from the wire.

a) By convention, since the step vectors are counter-clockwise around the loop, the direction of the area vector is determined by a RHR, so points out of the page, $\odot$. By the wire RHR, the magnetic field through the loop caused by the wire is into the page, $\otimes$. This means that for all $d\vec{A}$ tiles in the loop, the angle between $\vec{B}$ and $d\vec{A}$ is 180°. Now we can see that

$$\Phi_{\vec{B}} = \int_{d}^{d+W} \vec{B} \cdot d\vec{A} < 0.$$

b) Since the magnetic field created by the current in the wire, at a distance u from the wire is given by $\vec{B}_{\text{wire}} = \frac{\mu_0 I}{2\pi u}$, we can see that the magnetic field varies with u, so we must recognize that $\|d\vec{A}\| = dzdu$ and evaluate a double integral to find the flux through the loop:

$$\Phi_{\vec{B}} = \int_{d}^{d+W} \vec{B} \cdot d\vec{A} = -\int_{0}^{L} \int_{d}^{d+W} \frac{\mu_0 I}{2\pi u} dzdu = -\frac{\mu_0 IL}{2\pi} \int_{d}^{d+W} \frac{du}{u} = -\frac{\mu_0 IL}{2\pi u} \ln\left(\frac{d+W}{d}\right).$$

c) The current in the loop is given by

$$i = \frac{\mathcal{E}}{R} = \frac{-1}{R}\frac{d}{dt}\Phi_{\vec{B}} = \frac{\mu_0 L}{2\pi R} \ln\left(\frac{d+W}{d}\right)\frac{dI}{dt}.$$

d) I in the wire is decreasing
$\implies$ $\vec{B}$ through the loop is decreasing into the page
$\implies$ $\vec{B}_{\text{ind}}$ (in coil) increases into the page
$\implies$ i_{ind} (in coil) is clockwise (RHR).

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I have upheld the highest principles of honesty and integrity in all of my academic work and have not witnessed a violation of the Honor Code.