Odd cycle zero forcing parameters and the minimum rank of graph blowups

Jephian C.-H. Lin

Department of Mathematics, Iowa State University

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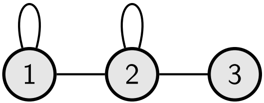
Discrete Math Seminar

Minimum rank problem (simple and loop)



$$\begin{bmatrix} ? & * & 0 \\ * & ? & * \\ 0 & * & ? \end{bmatrix} \qquad \begin{bmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

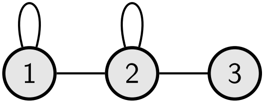
smallest possible rank
largest possible nullity

$\mathrm{mr}(P_3) = 2$ $\qquad$ $\mathrm{mr}(\mathfrak{P}_3) = 3$

$M(P_3) = 1$ $\qquad$ $M(\mathfrak{P}_3) = 0$

Zero forcing number (simple and loop)

$M(P_3) = 1$
$Z(P_3) = 1$



$M(\mathfrak{P}_3) = 0$
$Z(\mathfrak{P}_3) = 0$

minimum number of blue vertices
to force all vertices blue

simple If y is the only white neighbor of x and x is blue, then $x \to y$.

loop If y is the only white neighbor of x ~~and x is blue~~, then $x \to y$. (x, y are possibly the same.)

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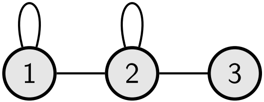

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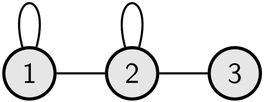

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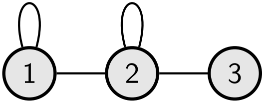

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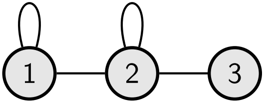

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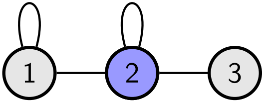

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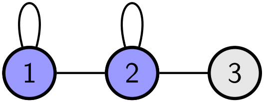

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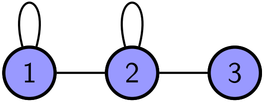

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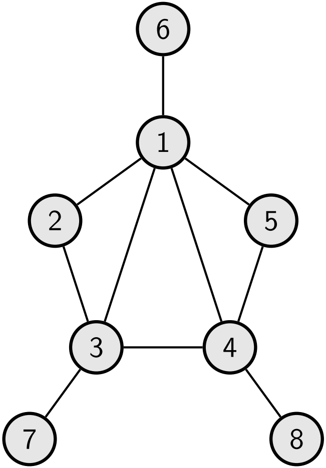
Max Nullity vs Zero Forcing

- $M(G) \leq Z(G)$ for all simple graph [AIM 2008]; $M(\mathfrak{G}) \leq Z(\mathfrak{G})$ for all loop graph [Hogben 2010].
- $M(G) = Z(G)$ whenever $|V(G)| \leq 7$ or G is a tree, a cycle; not always true for outerplanar graphs.
- $M(\mathfrak{G}) = Z(\mathfrak{G})$ whenever $|V(\mathfrak{G})| \leq 2$ or $\mathfrak{G}$ is a loop configuration of a tree; not always true for cycles or outerplanar graphs.

An example with $M(G) \neq Z(G)$



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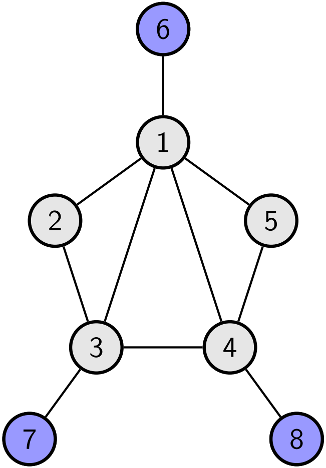

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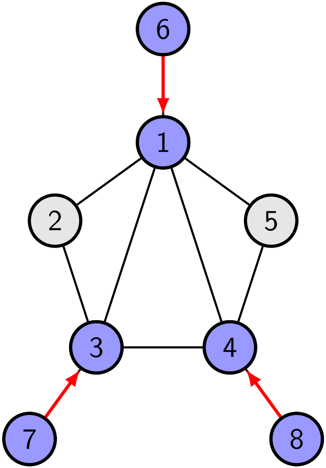

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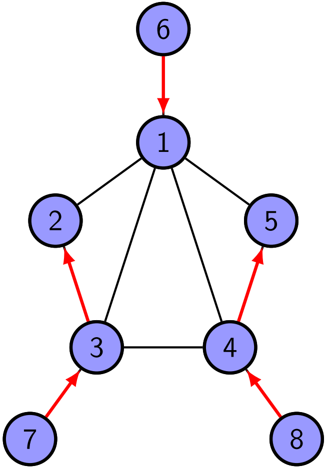

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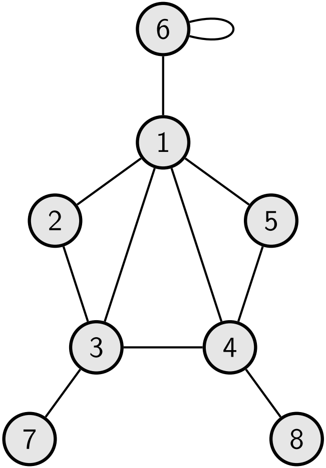

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others unknown

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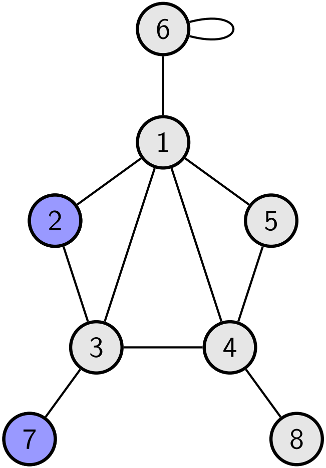

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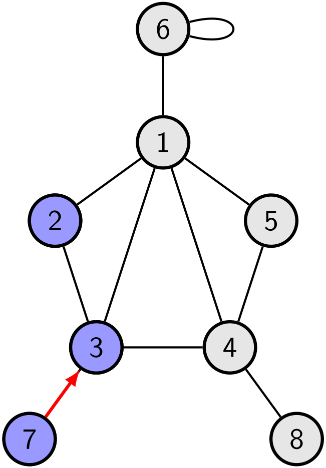

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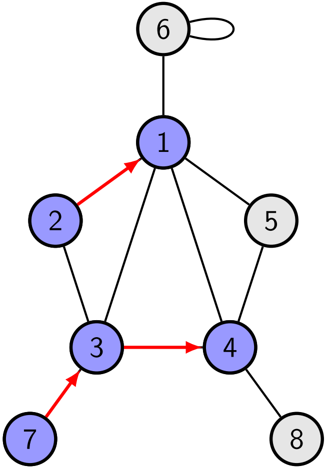

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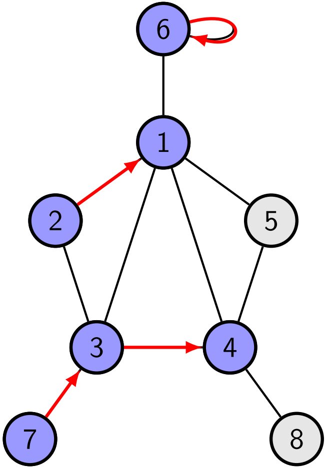

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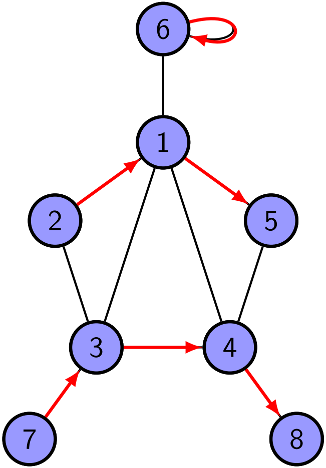

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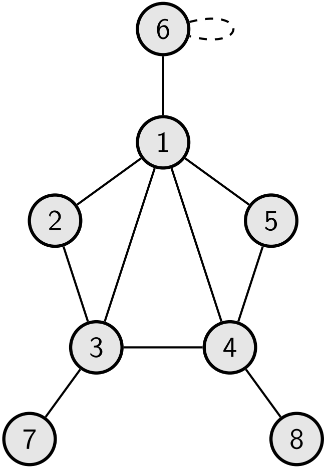

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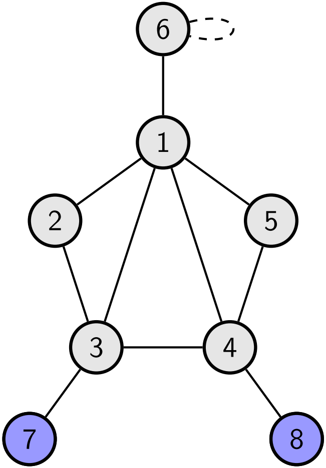

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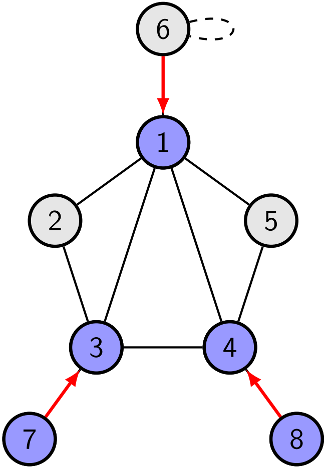

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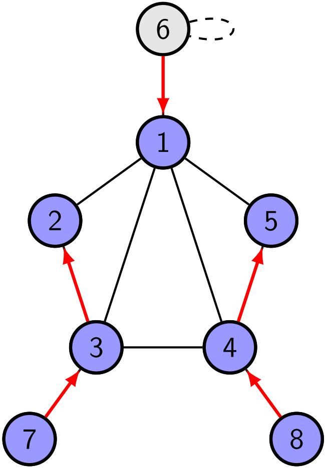

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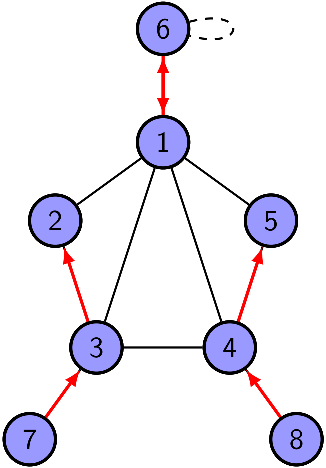

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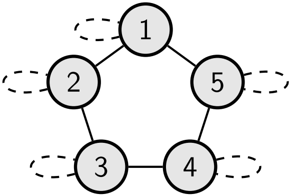
An example with $M(\mathfrak{G}) \neq Z(\mathfrak{G})$

- $M(\mathfrak{C}_n) = Z(\mathfrak{C}_n)$ if $\mathfrak{C}_n$ is not a loopless odd cycle; $M(\mathfrak{C}^0_{2k+1}) = 0$ but $Z(\mathfrak{C}^0_{2k+1}) = 1$.
- $M^{\mathbb{R}}(\mathfrak{C}^0_{2k+1}) = 0$ but $M^{\mathbb{F}_2}(\mathfrak{C}^0_{2k+1}) = 1$.



$$\det \begin{bmatrix} 0 & e_1 & & & e_{2k+1} \\ e_1 & 0 & e_2 & & \\ & e_2 & \ddots & \ddots & \\ & & \ddots & & e_{2k} \\ e_{2k+1} & & & e_{2k} & 0 \end{bmatrix} = 2 \prod_{i=1}^{2k+1} e_i$$

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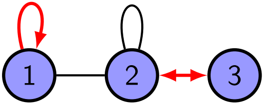
- For simple graphs with $|V(G)| \leq 7$, $M(G) = Z(G)$.
- For the simple graph $K_{3,3,3}$, $M(K_{3,3,3}) = 6$ and $\widehat{Z}(K_{3,3,3}) = 7$.
- For the loop graph $\mathfrak{C}_3^0$, $M(\mathfrak{C}_3^0) = 0$ and $Z(\mathfrak{C}_3^0) = 1$.
- The fact $M^F(\mathfrak{C}_{2k+1}^0) = 0$ is true whenever the considered matrix is symmetric and char $\neq 2$.

Proof of $M \leq Z$



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{bmatrix} \end{array}$$

$3 \to 2$
$1 \to 1$
$2 \to 3$

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Try to generalize the "triangle"

$$\operatorname{rank}\begin{bmatrix} a_{1,1} & ? & ? & ? & ? \\ 0 & a_{2,2} & ? & ? & ? \\ 0 & 0 & a_{3,3} & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix} \geq 3$$

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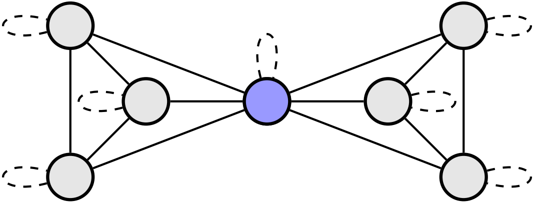
Try to generalize the "triangle"

$$\operatorname{rank}\begin{bmatrix} A(\mathfrak{C}_5^0) & ? & ? & ? & ? \\ O & A(\mathfrak{C}_7^0) & ? & ? & ? \\ O & O & A(\mathfrak{C}_3^0) & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix} \geq 5 + 7 + 3 = 15$$

Odd cycle zero forcing number

- The color-change rule for loop graphs is:
  - if y is the only white neighbor of x ~~and x is blue~~, then $x \to y$. (x, y are possibly the same.)
  - if W is the set of white vertices, and $\mathfrak{G}[W]$ has a connected component $\mathfrak{C}$ such that $\mathfrak{C} \cong \mathfrak{C}^0_{2k+1}$, then all vertices in $V(\mathfrak{C})$ turn blue.



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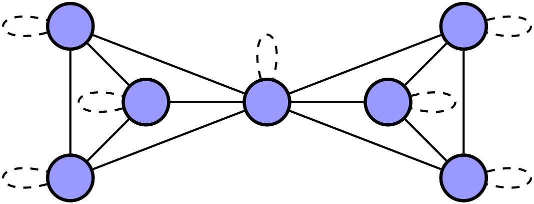

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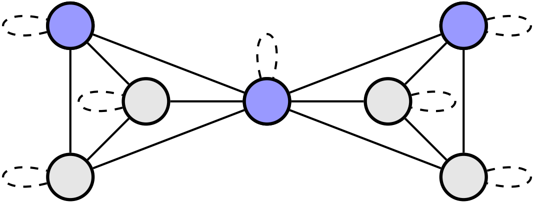

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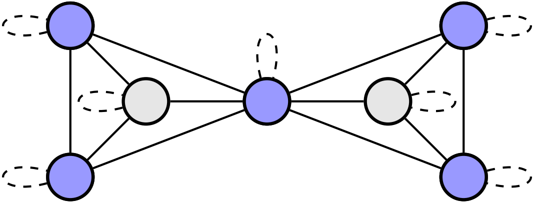

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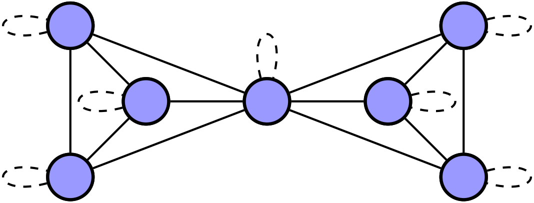

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  - if W is the set of white vertices, and $\mathfrak{G}[W]$ has a connected component $\mathfrak{C}$ such that $\mathfrak{C} \cong \mathfrak{C}^0_{2k+1}$, then all vertices in $V(\mathfrak{C})$ turn blue.
- $Z_{oc}(\mathfrak{G})$ is the minimum number of blue vertices required to force all graph blue.
- $M^F(\mathfrak{G}) \leq Z_{oc}(\mathfrak{G})$ whenever char $F \neq 2$ and matrices are symmetric.
- The enhanced odd cycle zero forcing number is defined as $\widehat{Z}_{oc}(G) = \max_{\mathfrak{G}} Z_{oc}(\mathfrak{G})$, where $\mathfrak{G}$ runs over all loop configurations of G.

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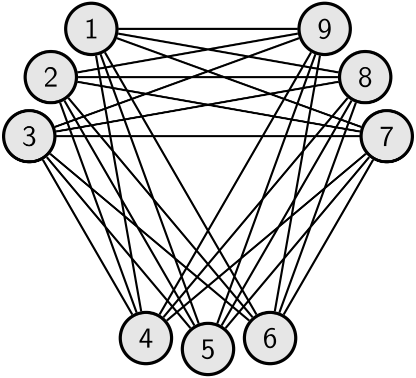
- For the loop graph $\mathfrak{C}_3^0$, $M(\mathfrak{C}_3^0) = 0$ and $Z(\mathfrak{C}_3^0) = 1$.
- $M(K_{3,3,3}) = 6 = \widehat{Z}_{oc}(K_{3,3,3})$.
- $M(\mathfrak{C}_{2k+1}^0) = 0 = Z_{oc}(\mathfrak{C}_{2k+1}^0)$.

Example: $K_{3,3,3}$



$$\widehat{Z}_{oc}(K_{3,3,3}) = 6 \text{ and } \widehat{Z}(K_{3,3,3}) = Z(K_{3,3,3}) = 7.$$

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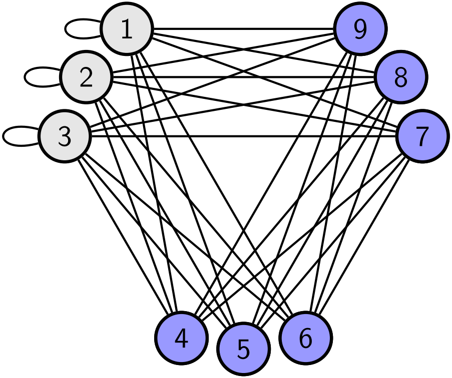

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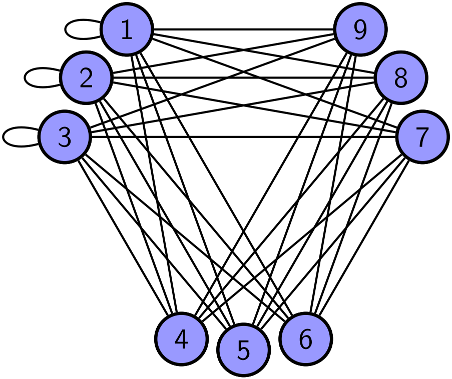
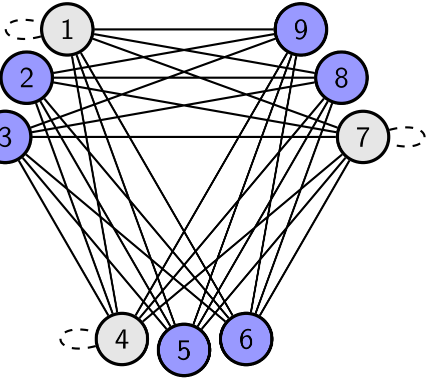


$$\widehat{Z}_{oc}(K_{3,3,3}) = 6 \text{ and } \widehat{Z}(K_{3,3,3}) = Z(K_{3,3,3}) = 7.$$

Example: $K_{3,3,3}$

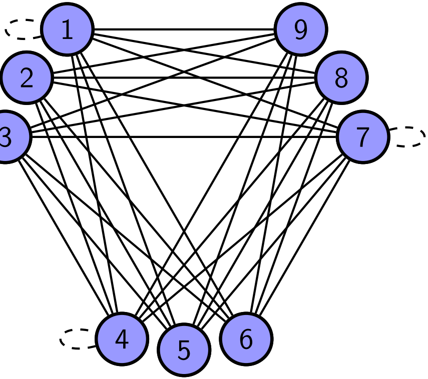
1,4,7 have no loops
others are unknown



$$\widehat{Z}_{oc}(K_{3,3,3}) = 6 \text{ and } \widehat{Z}(K_{3,3,3}) = Z(K_{3,3,3}) = 7.$$

Example: $K_{3,3,3}$

1,4,7 have no loops
others are unknown



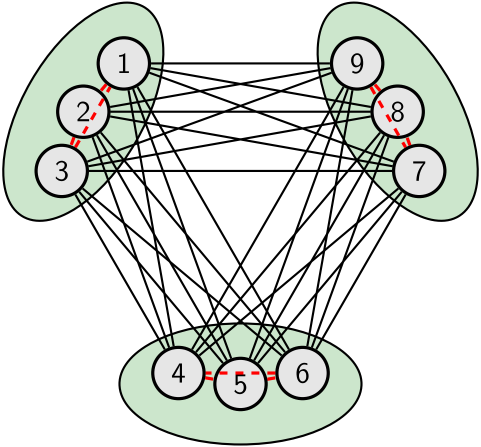
$$\widehat{Z}_{oc}(K_{3,3,3}) = 6 \text{ and } \widehat{Z}(K_{3,3,3}) = Z(K_{3,3,3}) = 7.$$

Field matters

- Let A be the adjacency matrix.
- $\text{null}(A) = 6 = M(K_{3,3,3}) = \widehat{Z}_{oc}(K_{3,3,3})$.
- $\text{null}^{\mathbb{F}_2}(A) = 7 = M^{\mathbb{F}_2}(K_{3,3,3}) = Z(K_{3,3,3})$.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

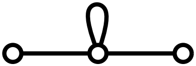
$\mathfrak{C}_3^0$ vs $K_{3,3,3}$



$\widehat{Z}_{oc}(K_{3,3,3}) = 6$ and $\widehat{Z}(K_{3,3,3}) = Z(K_{3,3,3}) = 7$.

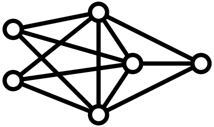
Graph & Matrix blowups

loop graph $\mathfrak{G}$



$\longrightarrow$ $(2, 3, 1)$-blowup

simple graph H



$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 7 \\ 0 & 7 & 0 \end{bmatrix} \xrightarrow{} \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 4 & 4 & 4 & 7 \\ 1 & 1 & 4 & 4 & 4 & 7 \\ 1 & 1 & 4 & 4 & 4 & 7 \\ 0 & 0 & 7 & 7 & 7 & 0 \end{bmatrix}$$

$(2, 3, 1)$-blowup

$A \in \mathcal{S}^F(\mathfrak{G})$ $\qquad$ $A' \in \mathcal{S}^F(H)$

Max Nullity vs Blowups

Lemma

Let the simple graph H be the $(t_1, t_2, \ldots, t_n)-$blowup of a loop graph $\mathfrak{G}$, and $\ell = \sum_{i=1}^{n}(t_i - 1)$. Then $\mathrm{mr}(H) \le \mathrm{mr}(\mathfrak{G})$ and $M(H) \ge M(\mathfrak{G}) + \ell$.

Proof.

- Let A be an optimal matrix for $\mathfrak{G}$.
- B, the $(t_1, t_2, \ldots, t_n)-$blowup of A, is a matrix in $\mathcal{S}(G)$.
- $\mathrm{rank}(B) = \mathrm{rank}(A)$.
- $\mathrm{mr}(H) \le \mathrm{rank}(B) = \mathrm{rank}(A) = \mathrm{mr}(\mathfrak{G})$.
- $|V(H)| = \sum_{i=1}^{n} t_i$, so $M(H) \ge M(\mathfrak{G}) + \ell$.

□

Graph Complement Conjecture

Corollary

Let $H \xleftarrow{(t_1,t_2,\ldots,t_n)} \mathfrak{G}$ with $t_i \geq 2$. Then $M(H) + M(\overline{H}) \geq |V(H)| - 2$, which means Graph Complement Conjecture is true for this H.

Proof.

- Since $H \xleftarrow{(t_1,t_2,\ldots,t_n)} \mathfrak{G}$, $\overline{H} \xleftarrow{(t_1,t_2,\ldots,t_n)} \overline{\mathfrak{G}}$.
- Recall $\ell = \sum_{i=1}^{n}(t_i - 1) \geq \frac{1}{2}\sum_{i=1}^{n} t_i = \frac{1}{2}|V(H)|$.
- $M(H) \geq M(\mathfrak{G}) + \ell \geq \ell \geq \frac{1}{2}|V(H)|$.
- $M(\overline{H}) \geq M(\overline{\mathfrak{G}}) + \ell \geq \ell \geq \frac{1}{2}|V(H)|$.
- $M(H) + M(\overline{H}) \geq |V(H)| > |V(H)| - 2$.

□

Zero Forcing vs Blowups

Lemma

Let $H \xleftarrow{(t_1,t_2,\ldots,t_n)} \mathfrak{G}$ *with* $t_i \geq 2$. *Then* $\widehat{Z}(H) = Z(H) = Z(\mathfrak{G}) + \ell$, *where* $\ell = \sum_{i=1}^{n}(t_i - 1)$.

Corollary

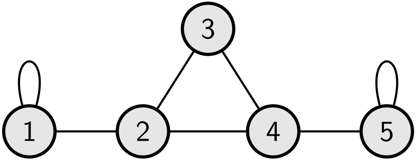
Suppose $H \xleftarrow{(t_1,t_2,\ldots,t_n)} \mathfrak{G}$ *with* $t_i \geq 2$ *and* $M(\mathfrak{G}) = Z(\mathfrak{G})$. *Then* $M(H) = \widehat{Z}(H) = Z(H) = M(\mathfrak{G}) + \ell$.

Proof.

- $M(H) \leq Z(H)$.
- $M(H) \geq M(\mathfrak{G}) + \ell = Z(\mathfrak{G}) + \ell = Z(H)$.

□

loop
rule



simple
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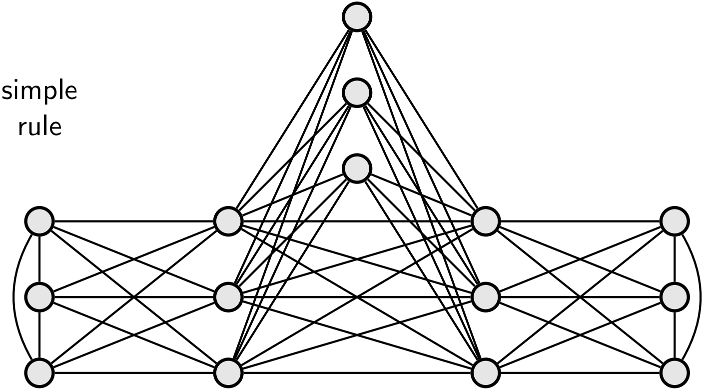

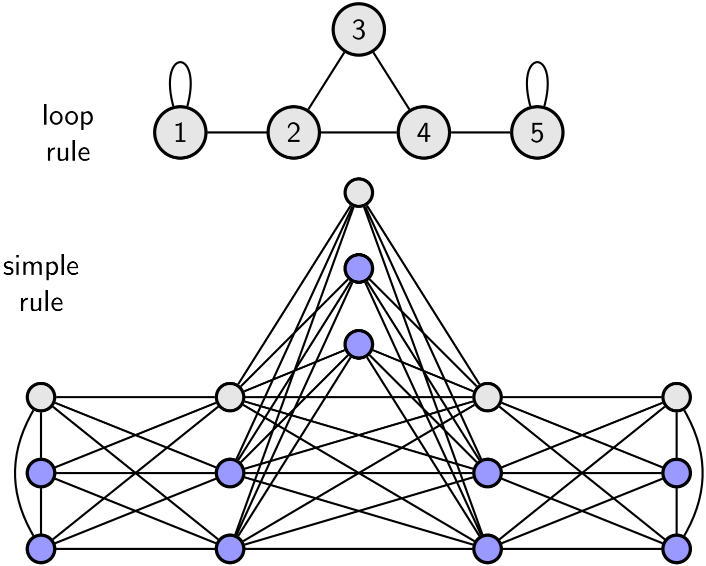
loop
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simple
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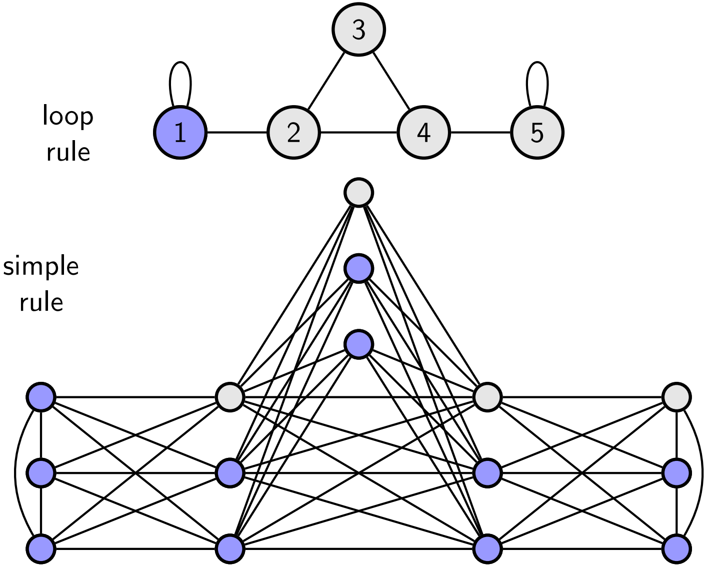
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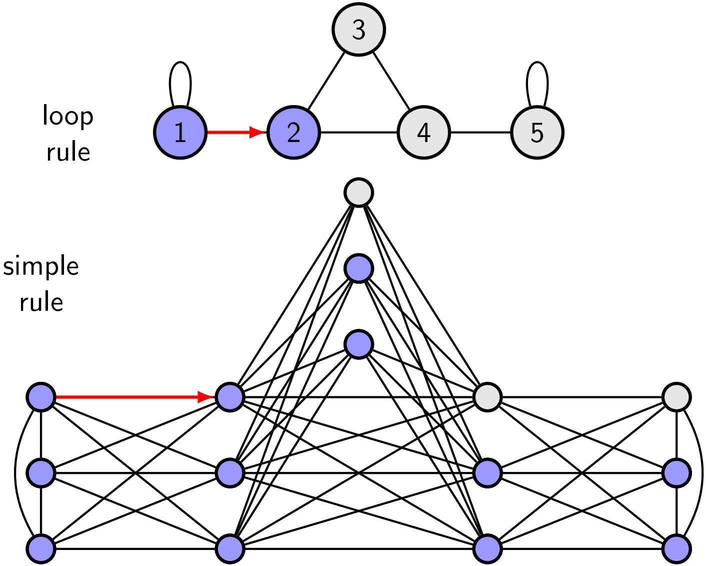

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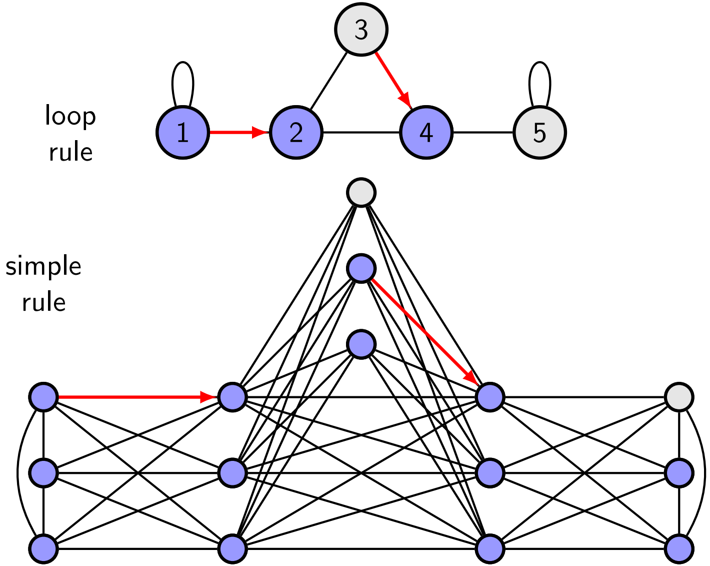
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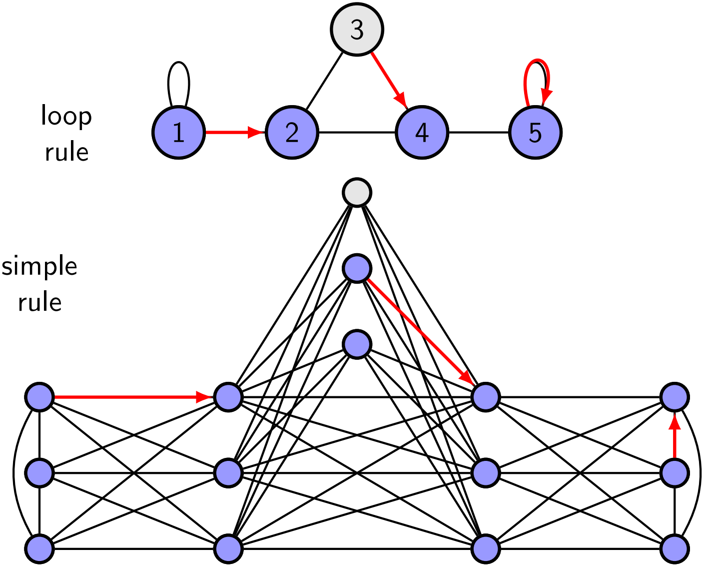

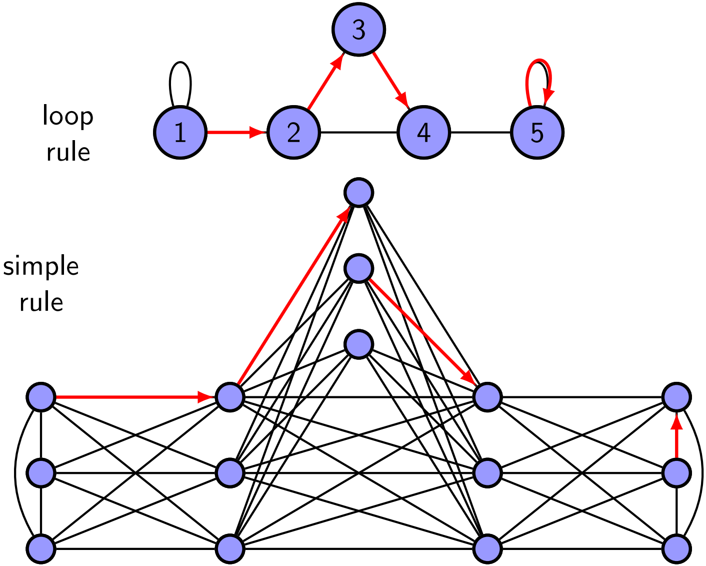
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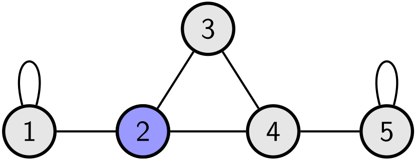
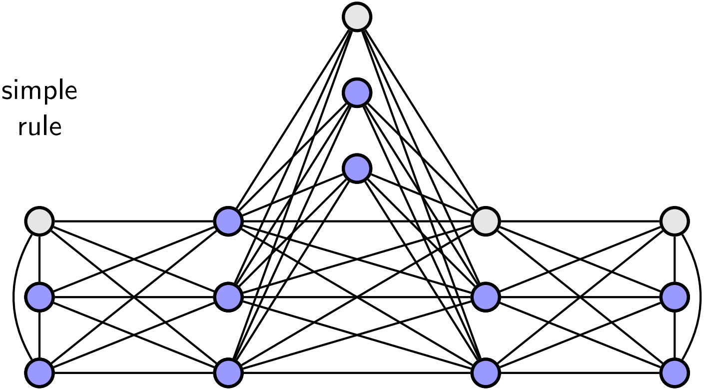
rule

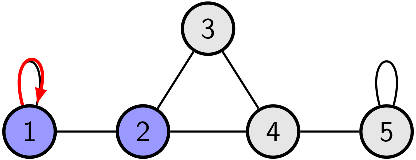
loop rule



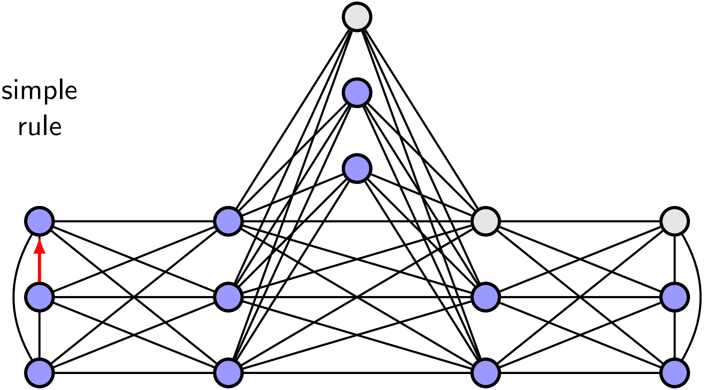
simple rule



loop rule

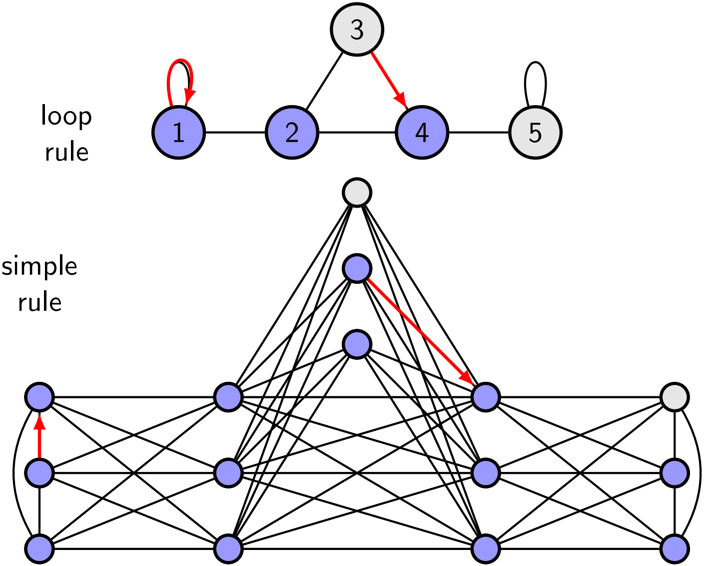


simple rule

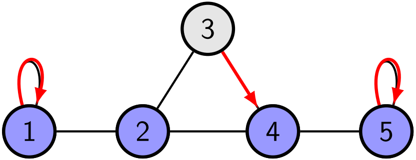


loop rule

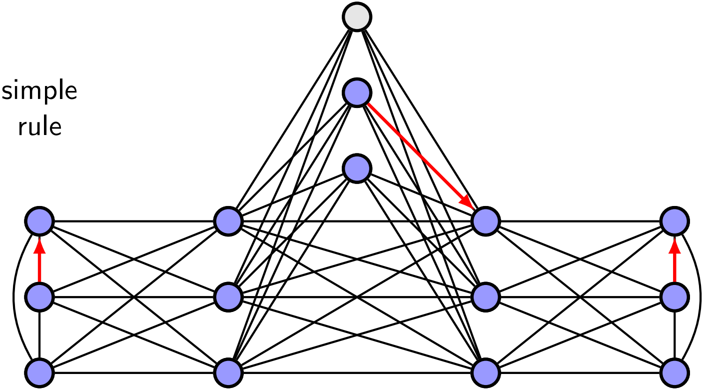
simple rule



loop rule



simple rule



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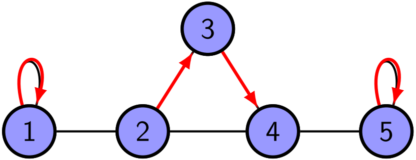
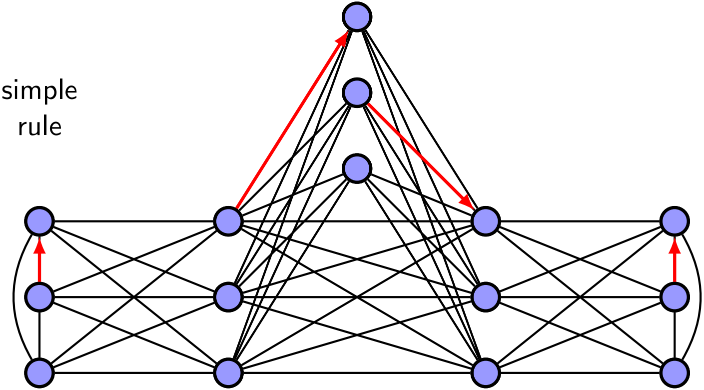


simple rule



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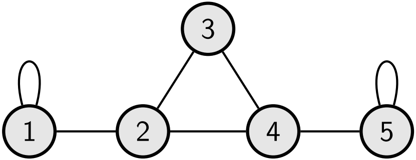

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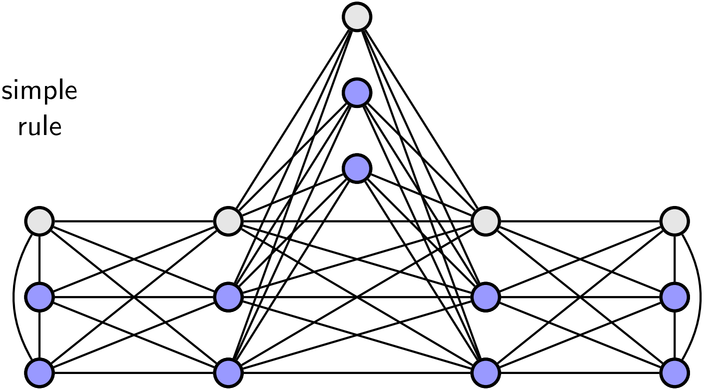


Zero Forcing vs Blowups Revisit

Lemma

Let $H \xleftarrow{(t_1,t_2,\ldots,t_n)} \mathfrak{G}$ *with* $t_i \geq 2$*. Then* $\widehat{Z}(H) = Z(H) = Z(\mathfrak{G}) + \ell$*, where* $\ell = \sum_{i=1}^{n}(t_i - 1)$*.*

Corollary

Suppose $H \xleftarrow{(t_1,t_2,\ldots,t_n)} \mathfrak{G}$ *with* $t_i \geq 2$ *and* $M(\mathfrak{G}) = Z(\mathfrak{G})$*. Then* $M(H) = \widehat{Z}(H) = Z(H) = M(\mathfrak{G}) + \ell$*.*

Proof.

- $M(H) \leq Z(H)$.
- $M(H) \geq M(\mathfrak{G}) + \ell = Z(\mathfrak{G}) + \ell = Z(H)$.

□

Odd Cycle Zero Forcing vs Blowups

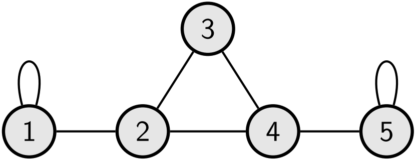
Lemma

Let $H \xleftarrow{(t_1, t_2, \ldots, t_n)} \mathfrak{G}$ *with* $t_i \geq 3$. *Then* $\widehat{Z}_{oc}(H) = Z_{oc}(\mathfrak{G}) + \ell$, *where* $\ell = \sum_{i=1}^{n}(t_i - 1)$.

Corollary

Suppose $H \xleftarrow{(t_1, t_2, \ldots, t_n)} \mathfrak{G}$ *with* $t_i \geq 3$ *and* $M(\mathfrak{G}) = Z_{oc}(\mathfrak{G})$. *Then* $M(H) = \widehat{Z}_{oc}(H) = M(\mathfrak{G}) + \ell$.

loop
rule



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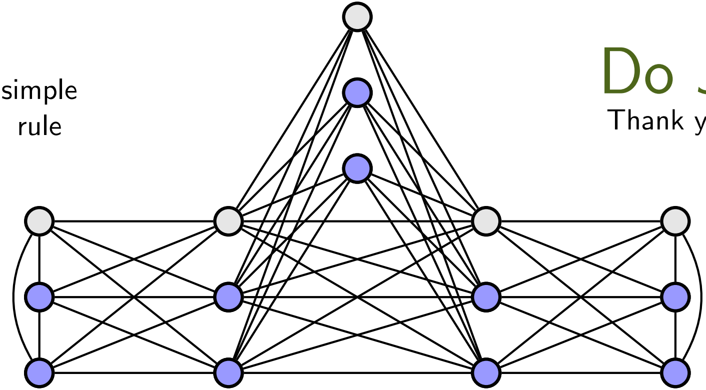

Do Je

Thank you.

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