First Meeting: 2-Body Problem

DCI

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Summary

In the problem of N bodies we start from the idea that each particle undergoes an acceleration due to the gravitational attraction of all the other particles in the system. This is expressed in a system of N coupled differential equations of second order.

$$m_i \ddot{r}_i = F_i = -\sum_{j \neq i} \frac{Gm_i m_j}{r_{ij}^3} r_{ij}, \quad i = 1, ..., N \tag{1}$$

where m_i, $_i$ and F_i are the mass, the position vector and the total force on the i-esima particle respectively, $r_{ij} = \vec{r}_i - \vec{r}_j$.

The resolution of the problem is completed by specifying the initial speeds and positions of the N particles. For resolution we divide the problem into two parts,

1. calculate the total force on each particle at a given time,

2. determine the new position and velocity of the particle in a new time $t+dt$.

In the method of direct integration, the force is calculated by solving the system of N equations (1).

One of the problems encountered is when the distance tends to zero, which means that the force diverges, appearing nonphysical accelerations in close encounters between particles. In a real physical system, this does not happen. To prevent this divergence is assumed that the particles are extended objects.

$$F_i = -\sum_{j \neq i} \frac{Gm_i m_j}{(|\vec{r}_i - \vec{r}_j|^2 + \epsilon^2)^{3/2}} (\vec{r}_i - \vec{r}_j), \quad i = 1, ..., N, \tag{2}$$

where ϵ is the length of smoothing or radio dilution (is the typical distance at which the interaction is altered).

Although ϵ avoids the divergent forces, when $r_{ij} < \epsilon$, will mean that the force is modeled not for the force of Newtonian gravity of a point mass. Hence the choice of the value thereof can not be arbitrary. For example, this approach is not correct for systems in which the value of ϵ is greater than the average separation of a significant fraction of particle pairs in the system.

1 Discussion on numerical methods

The first resolution method presented is to Euler method (which apparently can not be used when have many particles) also is commented the Runge Kutta 4th order method (RK4). For this case the error is on the order of the interval, h^5, and the accumulated error is to the order h^4. It is necessary to control the interval and for that is necesary use an adaptive RK4.