Independence Bounds Investigation

GBP Summer 2017 Working Group

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1 Goal

The **goal** of this project is to discover new bounds for the independence number of a graph, that is, bounds not implied by any other published bound: this means that the new bound must give a better value for at least one graph than any value given by any published bound.

Towards this goal we will code in all published bounds. The CONJECTURING program is designed to produce better bounds than any of the input theorems. The theorems will involve invariants we haven't seen before—-so these will also need to be coded.

The **dream** is to code all published graphs and invariants. In this case, any conjectures that the CONJECTURING program produces will be the *simplest* possible using concepts known to graph theorists (that is, published in the graph theory literature). By the design of the CONJECTURING program, the produced conjectures will necessarily be true for all graphs known to graph theorists (that is, published in the graph theory literature).

2 Invariant Definitions

Def. An *independent set* is a set of vertices which are pairwise non-adjacent (there no edges between any pair of vertices of the set).

Note: Finding a maximum independent set in a graph is *hard* (in fact its an NP-complete problem). All known algorithms are exponential in the number of vertices of the input graph.

Def. The *independence number* α of a graph is the cardinality of a maximum independent set.

Def. The *order* n of a graph is the number of vertices.

Def. The *size* m of a graph is the number of edges.

Def. The *degree* of a vertex is the number of edges incident to that vertex.

Def. The *minimum degree* δ of a graph is the smallest degree of any vertex of the graph.

Def. The *maximum degree* Δ of a graph is the largest degree of any vertex of the graph.

Def. The *neighbors* of a vertex v of a graph, denoted $N(v)$, is the set of vertices that v is adjacent to (connected by an edge to).

3 Theorems

All theorems are interpreted for connected graphs.

1. $\alpha \leq n$.
2. $\alpha \leq n - \delta$.

   Proof. Let g be a graph and let I be a maximum independent set. Let v be a vertex in I on smallest degree. Since no neighbor of v is in I. We have $\alpha = |I| \leq n - |N(v)| \leq n - \delta$.
3. $\alpha \geq \frac{1}{n}$.
4. $\alpha \leq n - radius$.

4 Conjectures

All conjectures are interpreted for connected graphs.

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