All work on this lab should be the collective effort of all group members. Technology allowed on this lab includes: Desmos (`https://www.desmos.com/calculator`) and an approved TI calculator. This lab has 3 questions for a total of 55 points.

1. For each of the following, find the derivative as indicated.

   (a) (5 points) $\dfrac{d}{dx}\left[\left(\dfrac{\sqrt{x}}{2}-1\right)^{-10}\right]$

   Solution:

$$\frac{d}{dx}\left[\left(\frac{\sqrt{x}}{2}-1\right)^{-10}\right] = -10\left(\frac{1}{\sqrt{x}}-1\right)^{-11}\left(\frac{\frac{1}{2}x^{-\frac{1}{2}}}{2}\right)$$
$$= -10\left(\frac{1}{\sqrt{x}}-1\right)^{-11}\left(\frac{1}{4\sqrt{x}}\right)$$

   (b) (5 points) $\dfrac{d}{dr}\left(\sqrt[3]{2r-r^3}\right)$

   Solution:

$$\frac{d}{dr}\left(\sqrt[3]{2r-r^3}\right) = \frac{1}{3}\left(2r-r^3\right)^{-\frac{2}{3}}\left(2-3r^2\right)$$
$$= \frac{2-3r^2}{3\sqrt[3]{\left(2r-r^3\right)^2}}$$

   (c) (5 points) Find y' if $y = \dfrac{1}{2x^7+3x}$

   Solution:

$$y = \left(2x^7+3x\right)^{-1}$$
$$y' = -\left(2x^7+3x\right)^{-2}\left(14x^6+3\right)$$
$$= -\frac{14x^6+3}{\sqrt{2x^7+3x}}$$

(d) (5 points) $\dfrac{d}{dx}\left(\log\left(3x^2-4x+1\right)\right)$

Solution:

$$\begin{aligned}\frac{d}{dx}\left(\log\left(3x^2-4x+1\right)\right) = \frac{d}{dx}\left(\frac{\ln\left(3x^2-4x+1\right)}{\ln(10)}\right) &= \\ &= \frac{1}{\left(3x^2-4x+1\right)\ln(10)}(6x-4) \\ &= \frac{6x-4}{\left(3x^2-4x+1\right)\ln(10)}\end{aligned}$$

(e) (5 points) $\dfrac{d}{d\theta}\left(\left(\theta^4-\sqrt{3-4\theta}\right)^8+5\theta\right)$

Solution:

$$\begin{aligned}\frac{d}{d\theta}\left(\left(\theta^4-\sqrt{3-4\theta}\right)^8+5\theta\right) &= 8\left(\theta^4-\sqrt{3-4\theta}\right)^7\left(4\theta^3-\frac{1}{2}(3-4\theta)^{-\frac{1}{2}}(-4)\right)+5 \\ &= 8\left(\theta^4-\sqrt{3-4\theta}\right)^7\left(4\theta^3+2(3-4\theta)^{-\frac{1}{2}}\right)+5\end{aligned}$$

(f) (5 points) Find $\dot{x}$, if $x(t)=e^{t^2+3t}-\log(2t)+t^2$

Solution:

$$\begin{aligned}x(t) &= e^{t^2+3t}-\log(2t)+t^2 \\ \dot{x}(t) &= e^{t^2+3t}(2t+3)-\frac{1}{\ln(10)t}\cdot 2+2t \\ &= e^{t^2+3t}(2t+3)-\frac{1}{\ln(10)t}+2t\end{aligned}$$

2. Consider the implicitly defined curve: $y^4 - 4y^2 = x^4 - 9x^2$.

(a) (5 points) Find $\frac{dy}{dx}$.

Solution:

$$\frac{d}{dx}\left(y^4 - 4y^2 = x^4 - 9x^2\right)$$
$$4y^3\frac{dy}{dx} - 8y\frac{dy}{dx} = 4x^3 - 18x$$
$$\frac{dy}{dx}\left(4y^3 - 8y\right) = 4x^3 - 18x$$
$$\frac{dy}{dx} = \frac{4x^3 - 18x}{4y^3 - 8y}$$

(b) (2 points) What is the value of $\left.\frac{dy}{dx}\right|_{(4,2)}$?

Solution:

$$\left.\frac{dy}{dx}\right|_{(4,2)} = \frac{4(4)^3 - 18(4)}{4(2)^3 - 8(2)} = \frac{23}{2}$$

(c) (2 points) What is the value of $\left.\frac{dy}{dx}\right|_{(2,4)}$?

Solution:

$$\left.\frac{dy}{dx}\right|_{(4,2)} = \frac{4(2)^3 - 18(2)}{4(4)^3 - 8(4)} = -\frac{1}{56}$$

(d) (8 points) Find the equation of the lines tangent to the implicit curve when $x = -3$. You will have exactly three lines for this problem.

Solution: To begin solving this problem, first the points where $x = -3$ must be found. By substituting $x = -3$ into the implicit function, the equation becomes $y^4 - 4y^2 = 0$. Solving this equation, we get the solutions $y = -2, 0, 2$. This means the function has the following points when $x = -3$: $(-3,-2), (-3,0), (-3,2)$. Now the slopes at each of the points must be found using $\frac{dy}{dx}$.

$$\left.\frac{dy}{dx}\right|_{(-3,-2)} = \frac{27}{8} \qquad \left.\frac{dy}{dx}\right|_{(-3,0)} = \text{undefined} \qquad \left.\frac{dy}{dx}\right|_{(-3,2)} = -\frac{27}{8}$$

Using these three slopes and the given points, we have the following lines

$$y + 2 = \frac{27}{8}(x+3) \qquad x = -3 \qquad y - 2 = -\frac{27}{8}(x+3)$$

$$y = \frac{27}{8}x + \frac{65}{8} \qquad y = -\frac{27}{8}x - \frac{65}{8}$$

3. If you drop a pebble into a large lake, you will cause a circle of ripples to expand outward. The area $A = A(t)$ and radius $r = r(t)$ are both function of t (they change over time) and are related by the formula $A = \pi r^2$.

(a) (4 points) If r is measured in inches and t is measured in seconds, what are the units of $\frac{dA}{dt}$? What are the units of $\frac{dA}{dr}$?

Solution:

$$\frac{dA}{dt} = \frac{\text{in}^2}{\text{sec}} \qquad \frac{dA}{dt} = \frac{\text{in}^2}{\text{in}} = \text{in}$$

(b) (4 points) Find $\frac{dA}{dt}$ and explain in pratical terms the meaning of $\left.\frac{dA}{dt}\right|_{t=2}$.

Solution: Using implicit differentiation, we can find that $\left.\frac{dA}{dt}\right|_{t=2} = 2\pi r \left.\frac{dr}{dt}\right|_{t=2}$. This means that at $t = 2$, the change in the area, with respect to time, is increasing at a rate of $2\pi r(2) \left.\frac{dr}{dt}\right|_{t=2}$.

Additionally, we can interpret $\left.\frac{dA}{dt}\right|_{t=2}$ as the direct proportional relationship between the radius and change in the radius with respect to time 2 seconds after the pebble was dropped into the water.