## 1 Introduction

The UTMOST Project (utmost.aimath.org) designs, develops and studies innovative approaches to the teaching and learning of undergraduate mathematics. We began by incorporating Sage, the powerful open source mathematical software system, into open electronic textbooks. Bringing together experienced undergraduate teachers, open textbook authors, mathematical software developers, and mathematics education researchers, UTMOST has produced curriculum materials, classroom resources, and computer-enhanced learning environments that address the needs of all the STEM disciplines and take advantage of recent technological progress in software and networks.

Motivation Motivating the UTMOST Project are two questions:

- How do students use textbooks?
- How can we develop textbooks that will improve teaching and learning?

These are fundamental questions for any subject at any level, but we focus on undergraduate mathematics courses with an emphasis on the incorporation of computation in the curriculum. Our focus is supported by the National Research Council [46] and the Mathematical Association of America [47], who have recommended that the undergraduate mathematics curriculum contains more computation and increases the amount of instruction on its use.

**Approach** Our approach to answering these fundamental questions, and to supporting computation in the curriculum, is to integrate computation directly into the everyday activities of the student and the instructor. We accomplish this through the creation of two new types of technology, both enabled by our prior work:

- Online textbooks that directly incorporate computation
- An online integrated collaborative computing environment, SageMathCloud

Both the textbooks and the collaborative computing environment will be available to the students in our test sites at no cost, from any connected computer, notebook, tablet or smartphone. Unlike most other approaches, there is no software for the student to install—thus significantly lowering the barriers to using computational tools for both students and faculty alike.

**Process** The textbooks and the collaborative computing environment will be hosted on servers at SageMath, allowing us to track all activity in the textbooks as well as nearly all student and instructor activity involving course material, with finely detailed time-stamped data and all tied to the individual. As far as we know, this will be the first time that it will be possible to directly monitor student use of textbooks in a manner that exactly captures the fine details of how, when, and for how long a student views a particular example, definition, or proof. The data collection will be unobtrusive and automatic, not relying on after-the-fact student reporting of activities. (We will also make use of other such data collection mechanisms. See Section 3.) We will be able to explore correlations between student activities, in particular the way they use their textbooks and the resulting learning outcomes.

**Components** Our project has several components that combine to address our motivating questions and assist with the measurement and study of our effectiveness.

- SageMathCloud (SMC): A robust, scalable web application that provides a collaborative scientific computing environment in the cloud. Users can collaboratively edit files of any type, use a command line terminal, chat or video conference, and interactively compute with Sage, Python, R, and other open source scientific computing tools. Instructors can easily assign, collect, grade and return assignments incorporating computation. With electronic textbooks also available as a SageMathCloud worksheet or a Jupyter notebook, this can become the electronic "home base" for a STEM course that incorporates computation.
- Sage Cell Server: This technology allows web pages, electronic textbooks, online homework, and other instructional materials to seamlessly embed live Sage computations, which can be edited by students.

- Online Open Textbooks: These textbooks exist in multiple electronic formats, and include Sage cells and other technology such as GeoGebra and WeBWorK. These are made possible by several results of our prior grant (Section 7): the American Institute of Mathematics (AIM) Open Textbook Initiative, the MathBook XML (MBX) flexible textbook format, and the Structured LATEX to XML (SL2X) program for converting existing open textbooks to our flexible format.
- Test Sites: Twenty-four courses at a diverse collection of institutions will provide a testing ground and produce data for our research component. There will be four pilot sites in Year 1 to help us refine our design and methods, followed by three semesters of data collection.
- Workshops: We will continue our annual Sage Edu Days workshops to foster and grow a community of users and developers of our resources and to provide formative feedback to the project as part of ongoing evaluation. An additional workshop will provide assistance to authors of open mathematics textbooks and lead to the conversion of a significant number of existing textbooks into our new formats.

What we have previously learned Our proposed work is made possible by our prior work, supported by an NSF CCLI Type II grant, which will enable us to create the necessary components and to use them for answering our motivating questions.

We developed an online version of PI Beezer's First Course in Linear Algebra (FCLA) textbook [8]. That effort led to understanding the process of converting an existing textbook written in LaTeX into a highly-functional online format. We began to develop SL2X, which is a tool that automates the conversion process, enabling the creation of online versions of a wide range of textbooks. This work also revealed many shortcomings inherent in the source structure of most textbooks, which make it difficult to convert textbooks to new digital formats for use on new devices. As a result we began developing MBX to serve as a format so that a single source can be used to easily convert to multiple formats—even those which do not yet exist. In Spring 2016 we will begin work on conversion to the popular EPUB format for e-readers. This one-time work will then enable any textbook in the MBX format to be output as EPUB.

Additionally, we converted PI Judson's Abstract Algebra: Theory and Applications (AATA) directly from LATEX to MBX, enabling its use online, and allowing the incorporation of significant material and exercises using Sage [31].

A major outcome of our previous grant is SageMathCloud. For the last several years, the primary way to interact with Sage has been through the Sage Notebook. This interface, similar to the notebook interfaces of Mathematica and Maple, was designed for a single user to develop worksheets of stand-alone series of computations accompanied by text and images. One of the outcomes of our previous grant was the ability to produce Sage worksheets from MBX source. These worksheets can function like sections of a textbook, and so they represent a step toward online textbooks incorporating computation. SMC has grown rapidly and is currently being used for a wide variety of courses—both traditional courses with a computational component and purely online courses. About 200 new accounts are created daily, and over 10,000 projects are modified in a typical week.

In our previous grant we constituted the AIM Editorial Board to develop objective standards for open textbooks and to apply those standards to available mathematics texts. The result was a list of 33 approved textbooks, covering 12 distinct undergraduate math courses. We have examined the source files of the approved textbooks and identified ten that are reasonable for us to begin converting. See Section 2.2 for more details.

Since most of those open textbooks have not been through the editorial process of a commercial publisher, the AIM Editorial Board (most of whom are published textbook authors) will also provide editorial guidance and an external critical review of the converted textbooks, in conjunction with incorporating Sage Cells and other enhancements throughout the documents.

The final ingredient which is made possible by our previous work is the Sage Cell Server, which

enables anyone to place Sage computations in web pages, and so, in particular, in our electronic textbooks. These Sage cells are live snippets of Sage code, which can be evaluated, edited, and reevaluated. Beezer's FCLA incorporates 579 Sage cells, primarily for examples and guided discovery, where the student does the computation *directly in the textbook* as an integrated activity while reading the textbook. Judson's AATA has 710 Sage cells. MBX provides an easy way to incorporate Sage cells into any online document.

# 2 Resource Design and Development

The open source resources under development by the UTMOST Project are indicated in Figure 2.1. These are each a direct result of our previous CCLI grant, and have seen active development since. We propose to further improve these resources to support our research component, and in reaction to the findings resulting from their use at our twenty-four test sites (Subsection 3.7).

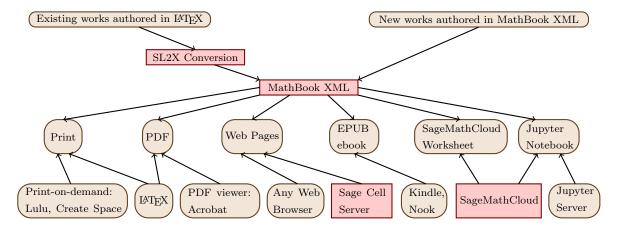


Figure 2.1: UTMOST Resources (Red Rectangles)

- Sage Cell Server: provides easy-to-use interactive Sage sessions in web pages
- SageMathCloud (SMC): an online collaborative computational environment for mathematics
- Mathbook XML (MBX): document source format, convertable to interactive output formats
- Structured Latex to XML (SL2X): conversion of legacy LATEX documents to MathBook XML

2.1 Sage, SageMathCloud, Sage Cell Server Sage, is a free, open-source, computer algebra system with capabilities similar to the commercial products Mathematica, Maple, MATLAB and Magma [47, Overview]. Since it is free to download and runs in a browser window, it is attractive to both students and faculty, especially at institutions with limited resources. The National Research Council's Mathematical Sciences in 2025 report concludes that the value of the mathematical sciences to the overall science and engineering enterprise of the nation would be heightened if there was an increase in the number of mathematical scientists that have some experience with computation [46, page 3]. The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics guide, 2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences, makes four cognitive recommendations, with one being "Cognitive Recommendation 3: Students should learn to use technological tools [47]. Mathematical sciences major programs should teach students to use technology effectively, both as a tool for solving problems and as an aid to exploring mathematical ideas. Use of technology should occur with increasing sophistication throughout a major curriculum."

Our software and textbooks are resources that make it easier for students to profitably learn and use Sage and other open source programs, and to make it easier for faculty to teach and administer a course that significantly incorporates computation.

SageMathCloud SageMathCloud (cloud.sagemath.com) is a major new online environment for collaboration among groups of people, such as a university class. It has Sage worksheets, powerful LATEX editing features, and a full Linux computer, all running in a standard web browser. Its main design feature is to enable and promote collaboration between groups of users. It is a natural place to host a course, allowing teachers to collaborate with their students using modern tools like Sage and LATEX. It has facilities for real-time communication through chat, video, and shared editing of documents, programs and Sage worksheets. Assignments can be distributed, collected, graded, and returned, all within SMC. With textbooks provided as Sage worksheets, SMC can provide all course materials required for a course.

SageMathCloud (SMC) is accessed via a web browser and requires no additional software. There are three principal arenas for collaboration. It is essential to realize that within any one arena, multiple users may *simultaneously edit and interact using their own independent cursor*. Each user sees every other cursor, annotated with the name of the cursor's owner. Side channels, such as a LATFX-enabled chat window, audio or video allow for traditional discussion. The three arenas are:

- Computational Worksheets. Sage worksheets are used to explore and annotate informal computational experiments and is a convenient format for instructors to distribute material. Jupyter notebooks similar, and supported fully within SMC.
- LATEX Documents. Students can construct careful proofs, analyze computational results, write short papers, term projects, or theses with professional tools needing no setup. SMC provides support for LATEX editing and compilation, and easy incorporation of results from Sage.
- Open Source Computing. A student's account in SMC may contain multiple "projects," which are full Ubuntu Linux computers, available online SMC. So additional open source computational software (R, Julia, Octave, Scilab, etc.) is installed and readily available, making SMC a platform for the full range of modern scientific computing tools, across all STEM disciplines.

SageMathCloud relies on continuous communication between a user's web browser and the cloud. So it is possible to track a student's time spent at various tasks such as viewing a textbook, using the interactive features of a textbook, working exercises, editing LATEX documents, discussing the course with a fellow student, or discussing the course with the instructor. We will implement the tracking and data collection features to provide "Instructor Analytics" in a dashboard identifying the overall progress of an entire class or of individual students. These data will also be stored and later analyzed in the course of our research investigations.

We will make use and enhance SMC's features in three principal ways.

- With our systems for producing open textbooks in a variety of electronic formats (see Section 2.2) we can convert textbooks to collections of SMC or Jupyter worksheets, including live Sage code, much in the same way that the Sage Cell Server enriches webpage versions.
- SMC provides excellent support for assignments in a course. Instructors can easily share worksheets with students in their course, collect assignments, and return graded work. We will further develop SMC to provide an even richer environment for managing a course, so that it becomes the "home base" for students outside of the classroom. Additionally, we will design an Instructor Dashboard as an analytical tool providing indicators of student engagement with course activities.
- With SMC's design, in which every action can be logged, it is possible to analyze a student's participation in various aspects of a course more precisely than has been possible before. We will have a significant amount of data for our research questions in Section 3, giving us a unique opportunity to quantify students' engagement with course material and activities, and to investigate correlation with course outcomes.

SageMathCloud has been employed in a diverse and expanding collection of courses the past three years. A representative list of over 170 such courses can be seen at [66]. SageMath, Inc provides free accounts to anyone. However, it is impractical to provide extensive free computing resources to the entire world and it is impractical to expect a large course to run on a free server. SMC's highly efficient design make it possible for individuals to run their projects on members-

only servers for under \$7 a month and receive a full complement of resources and network access. Incredibly, subscriptions for courses on members-only servers cost between \$1 and \$2 per student per month. At the end of a course, a student's project automatically moves to a free server, where a student can always access their work.

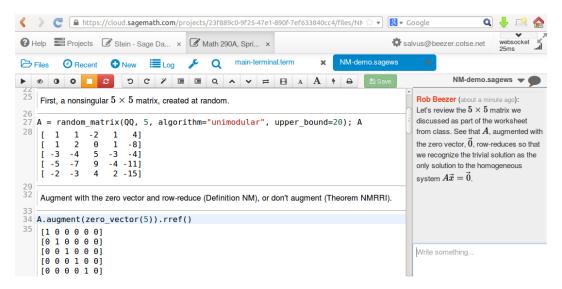


Figure 2.2: SageMathCloud: Two Sage cells and output, along with LATEX-enabled chat window

While the PIs and our motivating questions are all aligned with mathematics courses, all of SMC's principal features are useful for any STEM discipline. While the sciences and engineering have obvious needs for numerical computations and visualization, computer science is another discipline where SMC's features are extremely natural for students and instructors.

We have a very close relationships with the team at SageMath, Inc., especially since the CEO and Founder, Dr. William Stein, is on our advisory board and was a PI on our previous grant. SageMath, Inc. is a commercial company that provides hosting services and computational resources in a long-term, sustainable manner. But the underlying software is open source. So the improvements and features we make for undergraduate course use, the analytical tools to help teachers monitor their students' progress, and tools for our data collection and storage will all be publicly available.

Sage Cell Server The Sage Cell Server provides communication between a web page, or mobile application, with a running instance of Sage. This result from our CCLI grant has proved extremely popular.

We have pioneered the use of the Sage Cell Server in our two online textbooks [8, 31], both as a means to amplify the subject matter, but also as a way to introduce students to the power of exploring mathematics computationally. The Mathematical Association of America now publishes Smith and Moore's online-only calculus textbook, *Calculus: Modeling and Application*, and one of its distinguishing features is the extensive use of the Sage Cell Server [62].

The Sage Cell Server is central in the proposed work to design, test, and develop online textbooks. Senior Scientist Andrey Novoseltsev is the current volunteer maintainer of the Sage Cell Server and as a consultant will use our research findings to inform the design of technical improvements to this new infrastructure for integrating computation into the undergraduate STEM curriculum.

2.2 Online Open Textbooks Commercial publishers have little interest in producing electronic texts that are easily transportable across a variety of devices. They typically reserve or control the right to make copies, diminishing many of the advantages of an electronic format. Commercial e-texts often have licenses that expire at the end of the term, and when compared with

print editions that allow for resale, their price is not dramatically less [68,69]. Textbooks with open licenses can be distributed in electronic forms across the Internet for almost no marginal cost, making it possible for every student to have the required texts regardless of financial circumstances.

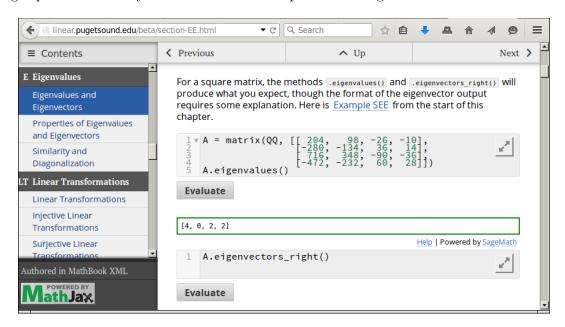


Figure 2.3: Executed Sage Cell embedded in online version of A First Course in Linear Algebra

It is too much to expect an individual author of an open textbook to master the technology to produce even a few of the existing formats. A major outcome of our previous CCLI grant was the conversion of open textbooks from L<sup>A</sup>T<sub>E</sub>X to electronic formats like webpages and Sage Notebook worksheets, with an emphasis on embedding interactive computational features. This experience has led to the development of two new important tools, based on the key recognition that any scholarly document is highly structured. If the *structure* of a document can be captured in the source format, then production in multiple formats can be achieved through simple software tools without requiring special technical expertise of the author.

SL2X is a Python program that analyzes a LATEX document to infer the structure the author intended. Then the content of the document can be converted to a format expressing that structure for use in subsequent processing. SL2X is under development now and can successfully convert many existing LATEX documents into web pages. This software greatly decreases the start-up cost to authors who wish to make use of our XML tools, described next.

MathBook XML (MBX, mathbook.pugetsound.edu) is a source format for authors that captures the structure of a document. For example, this proposal was authored in MBX (and edited collaboratively in SageMathCloud). The syntax is XML, a standard that is supported by powerful tools for conversion to other formats. The elements and attributes of the XML application are designed for ease-of-use by authors. A key feature is that it is easy for authors to embed interactive computational tools, such as Sage cells. We can convert MathBook XML into print, PDF, web pages and SageMathCloud worksheets, with full support for the demands of a mathematics textbook, such as typography, extensive numbering, and cross-referencing. Conversions to EPUB and Jupyter notebooks are at various stages of readiness, and will be completed during this project.

There are presently over a dozen textbook projects authored with MathBook XML, completed or well underway, including one for computer science. SL2X is retrospective, making existing works available in new formats. MathBook XML is prospective, enabling new works to take full advantage of multiple modern electronic formats. There are two goals for these tools. The first is to greatly increase the supply of open textbooks, providing enormous benefits to students and society. The

second is to provide textbooks that take advantage of new electronic formats in ways that increase a student's understanding and which we will investigate in our research.

We will base our research component on two very mature open mathematics textbooks with XML source formats. Beezer's A First Course in Linear Algebra debuted in 2004, and in recent years served as the prototype for the MBX and SL2X development efforts. When courses are in session, the online version attracts 2,000 readers a day. Judson's Abstract Algebra: Theory and Applications was published commercially in 1994 and provided with an open license in 2009. It was converted to Mathbook XML starting in May 2014. Since the online version debuted in August 2015 it averages 400 readers per day when classes are in session. Both have extensive interactive content using Sage. Dr. David Farmer will lead the development of SL2X, while Dr. Robert Beezer will lead the work on MathBook XML.

## 3 Education Research

The goal of our mathematics education research is to understand the affordances and challenges of using open source learning platforms, specifically SageMathCloud (SMC), by concentrating on the teaching two specific undergraduate mathematics courses: Linear Algebra and Abstract Algebra. This project examines the design enactment capacity of the resources available to undergraduate instructors via SMC and instructors' pedagogical design capacity when using SMC [13]. In parallel, the research examines students' use of SMC and how such uses contribute to their learning. Rather than bringing a fidelity perspective to the use of the resources available to the teachers [16] we approach the work with an enactment perspective [13, 14, 51] that attends not only to features of both teachers and resources that can influence instructors' practice, but also to the ways in which the use of resources can change personal features as a result of teacher inquiry into how students engage with the materials [14, 26, 27, 43]. We will investigate the work of instructors in planning and enacting curriculum embedded in SMC and the work of students as they use SMC to learn the material. We will contrast this with the use of other less dynamic resources (e.g., a "digitized version of traditional textbook," see [15]). We will focus on the following research questions:

- How do faculty use resources (SMC or less dynamic resources) in teaching? Specifically,
  - How do faculty use resources for planning lessons (prior to being in the classroom)?
  - How do faculty use resources during lesson enactment?
  - What changes or insights do they bring to teaching these courses in a new semester?
- What are the qualitative differences in resource use by faculty that teach these courses with SMC versus faculty who teach with less dynamic resources?
- What can explain the differences in these uses? For example: teaching experience, instructor experience with online resources, instructor attitudes towards technology, institutional characteristics.
- How do students use these resources for learning? Specifically,
  - How do students use the resources inside the classroom?
  - How do students use the resources outside the classroom?
  - How do such uses contribute or hinder their learning?
- What are the qualitative differences in what students learn when they are in a classroom that uses SMC compared to a class that uses less dynamic resources?
- Are there differences in learning gains that can be associated with the type of resource (SMC or less dynamic resources) used and what can account for the difference if they exist?

Together these questions help us understand curriculum design principles that best support teachers and students as they learn complex mathematics. Although this proposal takes into account the multiple users of the resources—authors, designers, instructors, and students—the research component focuses on the instructional aspect of the resource use, with the goal of informing the authors and designers of the open source textbooks.

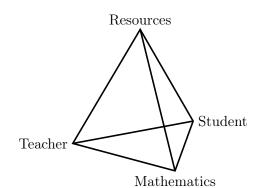
Dr. Vilma Mesa, assisted by a half-time graduate research assistant, will lead the educational research effort. Dr. Tom Judson also has broad experience with mathematics education research.

3.1 Rationale There are a number of claims about using "digital curriculum programs" regarding access, openness, and their transformative potential for teaching and learning, that need to be assessed in real classrooms [15]. While the potential for novel use is embedded in the design of many of these resources, it is not clear that instructors or students take full advantage of these capabilities, or which conditions lead instructors and students to be more likely to use the resources. In a study at a Norwegian university researchers report students' low use of the forum feature in online, distance learning mathematics courses [45]. The faculty speculated that the lack of a mathematical formula editor could be the reason for the low use. Yet, even though an improved editor with formula capabilities was added, the students did not take advantage of it. Why? Students did not have extra outside time to use the forum.

According to Kieran and colleagues, "resources are not neutral; they speak to different teachers in different ways — even to teachers using the same resources and sharing the same goals" [38, p. 211]. Teachers' personal and professional histories and experiences teaching a particular course, their beliefs about how resources should be used, and their goals, all shape what they can accomplish with the resources [13, 52, 58, 60]. And although one can pursue an analysis of the resources independently of the users, to "fully understand the role of curriculum materials... a comprehensive content analysis of [their use] by teachers can help to develop deeper knowledge of its functions in mathematics education" [27, p. vi]. To this, we add that an account of resource use that does not include the students is incomplete, given that the students are the ultimate beneficiaries of these new curricula [56]. Moreover, as part of a development project, the knowledge gained via a scholarly approach to resource use is vital for making appropriate research-based design adaptations. This study benefits from the large and extensive body of research on curriculum and technology use in K-12 settings and contributes to the more modest literature on resource use in undergraduate instructional settings (see e.g., [25, 26, 43]).

**3.2 Theoretical Background** At the core of this investigation is an interest in resources and their use for pedagogical purposes. The increasing range of textual and digital resources that are available to instructors has prompted empirical work on teachers' use of these resources that has led to a broader conceptualization of resources in mathematics teaching that includes teachers, their professional knowledge, and other material and cultural resources such as time and language [1]. In particular, Adler [2] has proposed the use of resource as verb, re-source, as 'taking-up,' a shift that helps moving the attention from the resources themselves refocusing "it on teachers working with resources, on teachers re-sourcing their practice" [3, p. 4]. A second important strand of work that informs this study is the documentational approach to resource use by instructors. Building on Vygotsky's triad of subject-task-tool [76], Rabardel and colleagues [49, 50] have proposed that human instruments have a dual character: "they contain components from artefacts themselves, and components from users' utilization schemes" [50]. In this conceptualization, one recognizes that users interact with their resources (in print or digital) in ways that are not anticipated by the designers. Instruments, for Rabardel, are artifacts accompanied by a scheme of use. A knife that can be used to spread butter or to lift the top of a bottle can be seen as two different instruments because of the different schemes of use that a user can give to the knife. Building on this work, Gueudet and Trouche [28] have proposed a theorization of how resources available to the teacher become documents for instruction through a cyclic process of instrumental genesis [50]: new documents become resources for a new process of document genesis, and this process evolves over time [43].

While most of the research on resource use concentrates on the instructors only, and specifically on K-12 teachers, due to their educative potential [16, 19, 28, 43, 53, 63, 71], Rezat [56] has argued about the importance of accounting for the student in such investigations, specially because students' work and thinking through, and with, the resources can shape what instructors eventually do in the classroom.



This is the third strand of work that informs this study. We adopt Rezat's didactical tetrahedron (Figure 3.1, [56, p. 241]) as a comprehensive model that helps in understanding the pivotal role of resource use in teaching. In the base of the tetrahedron, are elements of the instructional triangle [17], a definition of instruction as the interaction between teachers, the students, and the content, embedded within environments. In Rezat's model, resources are seen as an interdependent element that modifies such interaction. The separation of mathematics as an individual element is important for this study because the same mathematical topic

(e.g., span or ring) will be presented to the users (students and instructors) differently, in an open source environment (SMC) versus a less dynamic resource (e.g., a PDF of the textbook). The tetrahedron also acknowledges the work done with teachers' use of resources (the Teacher-Mathematics-Resources face) but highlights that how teachers interact with the resources to teach specific mathematics is necessarily informed by their interest on making the mathematics accessible to the students. An important contribution of this study will be the parallel work that can be conducted regarding students' use of the resources to learn mathematics (Students-Mathematics-Resources) and the interdependent way in which teachers and students interact with the resources. The literature on student use of resources is scant, limited to how German gymnasium students use their mathematics textbooks for studying [54, 55] or how college students report using their textbooks [78, 79]. Although initial conceptualizations of use have been proposed (e.g. [61, 77]) very little empirical work has been conducted (see also [59]). The findings of this project will make a significant contribution to understanding resource use by students, and instructors, in real mathematics classrooms.

3.3 Methods We propose an embedded mixed methods study [18] that gathers data on resource use by instructors and students in linear algebra and abstract algebra classes over three semesters. using either SMC or a less dynamic resource (e.g., a digitized version of the printed text), to contrast those uses, and to generate measures of student learning that would potentially identify the impact of the resources. Choppin [14] has advocated for longitudinal studies of teacher curriculum use to understand how such engagement evolves over time and what teachers learn about student learning through repeated use of the resources. We add that beyond learning about student learning, a longitudinal study of resource use allows designers to understand the ways in which instructors instrument their resources over time, as there are reports that they tend to rely less on resources as their familiarity with them increases [43], although this lower reliance does not mean that the instructors do not use them [39]. In embedded mixed methods designs the researchers collect qualitative and quantitative data simultaneously "so that one form of the data supports the other form of data" [18, p. 544]. The qualitative data is used to understand how instructors use the resources to plan and teach, and students to learn, linear algebra or abstract algebra. To accomplish this goal data will be collected at two different levels of detail, with instructors and their students providing biweekly reports on various aspects of their use of the resources. A group of four instructors and their students (each term) will provide additional in-depth data, including observation of planning and instruction, documentation of studying, interviews with faculty, and focus groups. The quantitative data will capture trends of such uses across the participants and investigate associations of these trends with student learning data. These forms of data are described in the next two section.

### 3.4 Data Collection, Basic Basic data collection for all courses will consist of:

- Instructor demographic and background information.
- Initial open-ended instructor surveys of their understanding of the course.
- Bi-weekly log of instructors' work in the course (resource use, content addressed, concerns).

- Student demographic and background information.
- Student surveys of resource use, perceptions of learning, attitudes towards technology.
- Student performance (start of the term): course-specific researcher-designed test of knowledge.
- Student performance (end of the term): course-specific researcher-designed test of knowledge.
- **3.5 Data Collection, In-Depth** Every term we will do in-depth data collection with instructors of four sections over a one-week period. The instructors will be followed over two or three semesters. Two sections (one linear algebra, one abstract algebra) will be using SMC, while the other two will be using less dynamic resources (one linear algebra, the other abstract algebra). We will use a form of the unit set methodology [14].
  - All of the basic information above (Section 3.4).
  - Video-recordings of instructors' planning and teaching sessions to capture thematically connected lessons (a sequence of two to five lessons).
  - Video-recordings of planning sessions and of lesson enactment, and document analysis for both instructors and students centered on a specific unit of instruction.
  - Interviews to establish instructors' views of the topic, goals, mathematical and pedagogical challenges, and key mathematical activities that are considered essential for student learning.
  - Event interviews to understand aspects of the instructors' work. The events are log entries and short video clips of planning sessions and lessons used to promote instructor reflection.
  - Focus group with students of these instructors will be conducted to identify patterns of resource use in- and out-side of the classroom.
  - Up to six students of these instructors will be asked to complete a detailed log recording how they use course resources both inside and outside of the classroom.
- 3.6 Data Analysis RQ1, How do faculty use resources (SMC or less dynamic resources) in teaching?, will be answered by determining the extent to which instructors offload, adapt, or improvise as they design and enact instruction [13] via an analysis of teachers logs and the individual, cross-case, and longitudinal-case study analysis. We will characterize the resources used by instructors and students depending on availability of multi-media (e.g., animations, graphics), interactivity with text (e.g., designing content), socialization (e.g., via blogs, chats), customization of learning experience (e.g., supporting students with additional needs), ongoing assessment of students' progress (e.g., automatic grading), cost effectiveness (e.g., faster and cheaper to revise), and accessibility (e.g., affordability, broadband access) as described in [15]. This characterization will be instrumental for differentiating how SMC and less dynamic resources are deployed by the users and will provide the context for differentiating use. This differentiation and the analysis resulting from RQ1 will help answer RQ2 (What are the qualitative differences in resource use by faculty that teach these courses with SMC versus faculty who teach with less dynamic resources?). The analysis of instructor characteristics collected via surveys, and the results of the analysis to RQ1 and RQ2 will help answer RQ3, What can explain the differences in these uses? To answer RQ4 (How do students use these resources for learning?) we will identify students' utilization schemes [57], differentiating by in-class and out-of-class use, and generating patterns that might be associated with their performance, using both their bi-weekly logs, and their responses as case participants. Linking these findings to those of RQ2 and the characterization of resources, we will be able to answer RQ5, What are the qualitative differences in what students learn when they are in a classroom that uses SMC compared to a class that uses less dynamic resources? Finally, RQ6 (Are there differences in learning gains that can be associated with the type of resource (SMC or less dynamic resources) used and what can account for the difference if they exist?) will capitalize on the findings from RQ1-RQ5 and use the student gain data using a two-level hierarchical linear model. The logistics for the qualitative data are as
  - The log responses, available as digital text, will be analyzed as soon as they are completed (every two weeks) using [13] and bottom up approaches that identify additional codes.

- Case study data (video recordings of planning and lessons, event interviews, focus groups, documents) will be coded digitally using standard qualitative analysis software (e.g., [30]):
  - Video recordings of planning sessions and lessons will be parsed into thematically coherent segments (e.g., task selection, time allocation, activity design for lesson planning; reviewing, lecturing, group work, for class observations; see [29, 42, 70]).
  - These segments will be coded for resource use with the top-down and bottom-up approach used to code the logs.
  - Cross-segment comparisons will be conducted to identify patterns of resource use and their connection to resource features to generate reports on individual case studies after each wave of data collection.
  - Individual reports will be used for cross-case analysis to identify salient differences across resource use by the type of resource (SMC or other less dynamic resources).
  - Longitudinal data will be carried out by individual cases and then by cross-case analyses.
- Documents will be tied to specific planning and lesson events and coded in terms of similarities
  and differences with provided resources and will be used as another source for evidencing
  change over time of the resources used.
- The quality of the teacher-student interaction will be used for characterizations of instruction, as has been done in studies of community college mathematics instruction [40, 42].
- Trustworthiness will be established via:
  - Triangulation: using the various types of sources (observations, reports in interviews, SMC data, student self-reports, surveys) to corroborate events and confirm use patterns.
  - $\circ$  Inter-rater reliability: by testing and refinement of the coding systems across raters (Mesa, her research assistant, and two of Michigan's Undergraduate Research Opportunity Program students) until Cohen's  $\kappa$  reaches 0.70 or more.
  - Member checking: the reports on use will be sent to the participants to seek their input regarding the validity of the interpretations that are made.
- Process auditing: our External Evaluator will provide auditing oversight of procedures. The logistics for the quantitative data analysis are as follows:
- The bi-weekly logs will be used to identifying frequency of resource use across time (within a semester) by instructor and by type of course [41, 44] and will be useful to identify resource use, both exemplified, and not exemplified, by the case studies. Aggregated data by course will be computed.
- Instructor surveys and background and demographic data will be aggregated and summarized across all participants.
- Student performance on researcher-designed tests of knowledge will be assessed by a rubric that will be fine-tuned during the pilot phase of the study; the coding will be subject to inter-rater reliability process as described above. With input from the advisory board, the PIs of the grant will design two tests, one for linear algebra and one for abstract algebra, with five questions each, to assess change in students' knowledge of major ideas and skills in each course (e.g., linear algebra: span, linear independence, bases; abstract algebra: equivalence classes, cosets, normal subgroups, ideals; proving and visualizing in both). The tests will be administered at the beginning of the course and one week before the end of the course, and a gain score will be calculated for each student.
- Student background and demographic data, and student self-reported learning gains will be summarized and linked to the learning gains data.
- Descriptive analysis (frequencies, correlations) will be used to determine how various student and teacher characteristics may be related to learning gains.
- A two-level hierarchical model (students, and students nested within instructors) will be used to identify significant associations between type of resource used (SMC vs. less dynamic resources) and student gains, both at the individual and at the classroom level. These will be moderated by the resource use variable generated through the log analysis.

All these are exploratory analyses meant to detect potential patterns of associations. Mesa and her research assistant, who will be proficient in statistical methods, will consult with U of Michigan's Center for Statistical Computation and Research office through the analysis and interpretation of the quantitative form of data. Table 1 describes the research component timeline.

**3.7 Test Sites** We will work closely with, and extensively support, twenty-four courses that will make our software and materials a central part of their courses. From the students in these courses, and their instructors, we will be able to collect a significant amount of data, much of it in an automated fashion through SMC. Four additional pilot sections in the first year of the project will allow us to refine the design of our investigations.

We solicited interested faculty through a variety of avenues, such as the communities for Project NExT, MAA Research in Undergraduate Mathematics Education SIGMAA, Inquiry-Based Learning, and Sage. We received applications from thirty schools, representing fourty-two faculty. From these we have selected twelve schools and sixteen faculty, seeking diverse representation of students, faculty, and institution character (public, private, religious), enrollment, and geography, while balancing our work across time. We have balanced instructor experience with computation, Sage and SageMathCloud, along with experience with the course material itself. We have not assigned five sections (even though we had many qualified applicants), to allow us the greatest possible flexibility to fine-tune our selections once we get closer to implementation. In our supplemental materials we have included, for each of the twelve schools, full details and a departmental commitment letter, along with an "at-a-glance" summary table.

We have some statistics for the more diverse student populations at our test sites: California State U, Long Beach: Hispanic Serving Institution; Mississippi C: 43% non-white; Sam Houston State U: 19% African-American, 18% Hispanic; State U of New York, Geneseo: math majors are 6% of undergraduates, 154 female, 163 male; Texas A&M U, Central Texas: 100% community college transfers, 40% military or military dependents; U of Michigan, Dearborn: 32% non-white.

#### 4 Dissemination

4.1 Sage Edu Days and Open Textbook Editorial Workshops Four Sage Edu Days Workshops have been held June 2011–2014 at the University of Washington, supported by our CCLI grant [72, 73, 74, 75]. These professional development workshops have proved a popular and productive way to exchange and refine ideas, in addition creating a community of supportive colleagues among faculty directly involved at our test sites, faculty using Sage and electronic textbooks in their classrooms, and faculty new to Sage. They have given the UTMOST PI's both constructive suggestions for improvements and the inspiration to innovate. Conversely, they are an excellent vehicle for disseminating our work and findings. We are requesting support to double attendance at these events.

A primary goal of our textbook authoring tools is to increase the supply of available open textbooks. Another professional development workshop in Summer 2018 will introduce new authors to these tools and assist them with the creation of highly functional online versions of their textbooks. This workshop will be led by the AIM Editorial Board, authors familiar with our tools (experts), and UTMOST PIs. A similar workshop in April 2016, funded by the American Institute of Mathematics, will include four of the UTMOST PI's and so give us valuable experience organizing such a workshop. Combining textbook content vetted by the AIM Editorial Board with improvements in our tools derived from test site feedback, this workshop will produce many open textbooks in highly functional formats.

Ms. Lynds, our External Evaluator will provide comprehensive external review for each of the four workshop as described in Section 5. Dr. Theron Hitchman will assist with the planning and execution of the workshops as a Senior Scientist.

**4.2** American Institute of Mathematics Open Textbook Initiative The American Institute of Mathematics' Open Textbook Initiative (aimath.org/textbooks) provides trusted guidance on the quality of freely-available undergraduate mathematics textbooks. The Editorial Board will expand their role in this project, using board members to provide editorial advice and guidance for authors of open textbooks.

Over the past five years, the AIM Editorial Board has evaluated more than 50 open source/open access textbooks for undergraduate mathematics courses and found 33 of them to recommend for adoption as required texts. All these textbooks are available free as PDF files and about half are also available for nominal cost in print editions (from \$10 to \$30). Only two, Beezer's linear algebra text, and Judson's abstract algebra, are available in a webpage version with the dynamic interactive features we are developing for online texts.

The Editorial Board, along with project PIs, will work with the most promising textbooks to develop fully functional online versions and print versions. Our goal is to provide open source/open access textbooks of the highest quality that are easy to use in all formats and on all common electronic devices. These features are not possible in commercial texts because of the obstacles caused by cost and licensing restrictions. In addition, a recent study [21] reports that students appreciate the portability and availability of electronic texts, but their "frustrations using their devices to access electronic textbooks outweighed their appreciation."

AIM Editorial Board members, all of whom have years of experience in the classroom and as authors, will work closely with the authors of selected texts to achieve our goal. Besides the dedicated workshop, authors and editors will collaborate through email and other web services, both before and after the workshop. Dr. Kent Morrison will continue to lead the AIM Open Textbook Initiative, and its Editorial Board.

**4.3 Other Dissemination Efforts** We will continue to disseminate project news, results and products through the central project website (utmost.aimath.org), in addition to utilizing existing websites for SageMathCloud, Sage Cell Server, MathBook XML, and the AIM Open Textbook Initiative [4, 7, 22, 64], in addition to online forums and workshop-specific sites. Code for Sage, Sage Cell Server, SageMathCloud, MathBook XML, A First Course in Linear Algebra, and Abstract Algebra: Theory and Applications are distributed with open licenses and hosted publicly on GitHub, with SL2X joining soon. So anyone can learn from the code, contribute improvements or distribute modified versions. Data and findings from the educational research activities will be published rapidly and openly, suitably anonymized, and as appropriate, with permission.

#### Research: Education Research Program

Education Research Program—Year 1 Pilot (4 courses), Years 2 and 3 full implementation (24 courses)

- Test-Site Research Model
  - Linear Algebra Courses with online text, SageMathCloud and Sage Cell Server implementation
  - Linear Algebra Courses with less dynamic resource
  - Abstract Algebra Courses with online text, SageMathCloud and Sage Cell Server implementation
  - Abstract Algebra Courses with less dynamic resource
- Test-Site Observations by research team
- Analysis of SageMathCloud usage data with enhanced course learning data
- Interviews with test-site faculty and students



#### **Evaluation: Evaluation and Critical Review**

- External evaluator review of research instruments and framework
- Annual formative evaluation reports (test-site faculty interview and SageMathCloud usage data)
- Advisory board annual recommendations

## Resource Design and Development: Online Texts, Sage Cell Server and SageMathCloud

For linear algebra and abstract algebra courses

- More formats for SL2X and MathBook XML outputs
- Improvements in Sage Cell Server stability, scalability, and hosting
- Enhance SageMathCloud for data collection (Instructor's Dashboard)



#### Evaluation: Evaluation and Critical Review

- Guidance from Advisory Board
- Critical review by Editorial Board
- Annual evaluation of SageMathCloud usage analytics by external evaluator

#### Dissemination: American Institute of Mathematics Open Textbook Initiative

- Editorial Board (experienced authors and editors)
- List of approved textbooks
- Editorial and practical guidance for open textbook authors



#### Evaluation: Evaluation and Critical Review

- Front-end evaluation registration surveys to maximize effectiveness of workshops and meetings
- Formative evaluation daily surveys and observation data to enhance workshops in real time
- Summative evaluation reports, including pre/post analysis and recommendations for next event



#### Dissemination: Workshops and Professional Development

- Sage Edu Days Workshops (in years 2, 3, and 4)—professional development for test site faculty and other stakeholders
- Author/Editor Workshop (in year 2) for online textbook authors and developers

Figure 4.1: Project Logic Model

### 5 External Evaluation

**5.1 Program Evaluation and Critical Review** Critical external review, front-end, formative, and summative evaluation will be conducted by Ms. Susan Lynds of the Education and Outreach group of the University of Colorado's Cooperative Institute for Research in Environmental Sciences (CIRES). In Year 1, Ms. Lynds will obtain Human Subjects Research approval from the University of Colorado's Office of Research Integrity for the evaluation work.

Critical Review Building on her broad expertise and in-depth experiences in evaluating and STEM programs, Ms. Lynds will provide critical review of the project's design and activities, including, but not limited to, the project activities, the research instruments, and the reports of findings to funding agencies, in presentations, and in journal articles. The evaluator will synthesize results from the front-end, formative, and summative evaluation at regular intervals, including overall recommendations to the management team via written reports; these activities are detailed in the evaluation plan below.

In addition, the management team will benefit from additional critical review and guidance on the project's theoretical framework and operational design by the advisory board and the editorial board, both of which are described in the proposal text.

These external critical reviews will be used to influence the project's activities, to ensure that project goals are met, and to improve the quality of the project findings. The evaluator will provide a summary of evaluation results for inclusion in each annual NSF report.

**External Evaluation** The evaluation is aligned with the proposed goals and objectives for the project. The evaluator will work closely with the researcher and the project management team to inform the project activities, including the research and development components, the advisory board and editorial board meetings, the test-site participants' online forum, the Sage Edu Days and author/editor workshops, and the use of SageMathCloud (SMC).

At the end of each implementation year, Ms. Lynds will conduct telephone interviews with test site (research participant) instructors to collect qualitative data on the project's goals and objectives. These data will be integrated with the SMC analytics data in order to provide a formative evaluation report after each year of implementation. This regular independent reporting will provide necessary input to influence the project's activities and improve the quality of its findings. All evaluation data will be shared with the researcher and the project management team.

Ms. Lynds will provide comprehensive external evaluation for the project's three Sage Edu Days workshops and the one Author/Editor workshop. The Sage Edu Days workshops will include test site representatives and a broader array of faculty involved with the project initiatives. The Author/Editor workshop will include open source textbook authors and editors. Ms. Lynds will provide independent evaluation of the workshops through customized surveys, informal conversations with participants, observation of the workshop activities, and daily meetings with the PIs as a group. Registration surveys for each workshop will provide front-end evaluation data to allow workshop facilitators to customize each workshop to meet the needs of the participants. Daily feedback survey data and observation data will be summarized for PIs each evening to allow for fine-tuning of the workshop in real time (formative evaluation). Summative evaluation surveys for each workshop will provide data for analysis of the overall effectiveness of each event. Post-workshop survey data will be paired with registration data to document changes in attitudes and knowledge levels by participants. A workshop evaluation report for each event will include recommendations for future workshops in the project.

Ms. Lynds will be a member of the project's online test-site participant forum; she will observe the support and interaction that occurs there throughout the year. Twice a year, evaluation reports to the team will assess the effectiveness of the forum.

Each year, annual online surveys will be conducted with advisory board and editorial board members after their annual meetings; the evaluator will attend these meetings virtually to collect observation data. Each year, these evaluation data will be analyzed and summarized in a formative evaluation report to the team so that each year's meetings may be improved.

Ms. Lynds will provide critical review of the educational research instruments as well as reports, presentations, and articles for online journals.

In the final year, the evaluator will interview all team members and all test-site faculty members (research participants) to gather their reflections on the project overall for inclusion in the final summative evaluation report. This report will also summarize the evaluation data collected throughout the project and will provide recommendations for future projects in the field.

The evaluator's office at CIRES has the tools, experience, and staff to provide efficient and professional assistance with evaluation activities, and can provide post-processing of data such as interview transcription. Other data on faculty student activities will be collected in SMC's automated analytics tools.

Amanda Morton will provide administrative support in the coordination of the student support work for Ms. Lynds in this project. Ms. Morton will also coordinate the distribution of Amazon cards to the test-site faculty who participate in interviews.

**Data Collection** Appropriate survey items and interview protocols will be developed and refined throughout the project. A mix of quantitative and qualitative evaluation methods will be used. Qualitative data include instructor, advisory board, and project team interviews, open-ended survey items, and observations of workshops. Quantitative data include multiple-choice survey items for workshop participants, advisory board members, and project team members, as well as SMC analytics data.

Data Analysis and Reporting Interview data will be recorded and transcribed. Interview and open-ended responses will be coded using a domain analysis [48]. Descriptive statistical analysis will be conducted on the data. Front-end, formative, and summative evaluation results will be compiled in reports for the project management team and for the annual reporting to the funding agency.

**5.2** Advisory Board An advisory board will provide additional critical review for the project, in addition to being available to provide advice on aspects of the project. Board members, who have agreed to serve in this capacity, are Dr. Jason Grout (Bloomberg), Dr. Kiran Kedlaya (University of California, San Diego), Dr. Gavin LaRose (University of Michigan), and Dr. William Stein (University of Washington). A short biography of each can be found in Section 8.

Grout, Kedlaya, and Stein were all PIs on our 2010-14 NSF CCLI project, and so are already very familiar with our work. Additionally, Stein is the creator of Sage and SageMathCloud, and CEO of SageMath Inc., while Grout is the creator of the Sage Cell Server. LaRose is new to the project and brings expertise in the study and evaluation of the use of technology in learning and teaching mathematics. The knowledge and experience of these individuals will be very beneficial to the project.

The board will be encouraged to use SageMathCloud in their work, will be invited to project workshops, and will participate in an annual meeting with project PI's to discuss their review of the project's activities, direction and findings.

	Year 1	Years 2 & 3	Year 4		
Resource Design and Development	SL2X MathBook XML Sage Cell Server SMC: course analytics	SL2X MathBook XML Sage Cell Server SMC: course support	SL2X MathBook XML Sage Cell Server Instructor Dashboard		
Education Research	Fall: research design Fall: instruments Spring: 4 pilot courses	Fall: 8 test courses Spring: 8 test courses Fall: 8 test courses Spring: data analysis	Data analysis Publish findings		
Dissemination	Sage Edu Days 7 Websites, Forums AIM Editorial Board	Sage Edu Days 8 & 9 Author Workshop Websites, Forums AIM Editorial Board	Publications Websites, Forums AIM Editorial Board		
Evaluation	Formative Evaluation Advisory Board	Formative Evaluation Advisory Board	Formative Evaluation Final Interviews Summative Evaluation Advisory Board		

Table 5.1: Project Timeline

# 6 Broader Impacts of the Proposed Work

The UTMOST Project creates infrastructure to support increased understanding of mathematical and computational topics in the undergraduate STEM curriculum. Through our educational research, test sites, workshops, and websites, we will learn about, and advise others, on best practices for use of our products.

Our products are delivered with open licenses, or are built on open technology, so they are free or very low cost. Textbook prices rose 82% from 2002 to 2012, according to a recent GAO report [20]. Free or low-cost textbooks, and highly efficient, reasonably priced computational resources can together make a critical difference in a student's success when the cost of an education is a barrier, or in the success of an institution with limited resources. We anticipate that a cost reduction will contribute to an increase in the participation of low-income and underrepresented minorities in STEM disciplines.

SageMathCloud enhances the infrastructure for education and research. It enhances STEM education by creating a collaborative learning environment that is specifically designed to support courses with a significant mathematics or computational component. Requiring no more than a login and a web browser, it extends the power of Sage cells with robust file storage in the cloud, processing power, long-form Sage worksheets, LATEX editing, and a full suite of open source scientific computing tools, all tightly integrated with modern tools for real-time communication and collaboration. The ability to capture fine-grained measurements of student activity provides exciting new possibilities for understanding student behavior, both in educational research and as guidance for instructors of courses.

Sage cells enhance the infrastructure for education and research. They are transforming materials used by students in STEM courses in novel and innovative ways. The Sage Cell Server provides

the full computational power of Sage embedded in webpages or applications, requiring absolutely zero set-up by users. It powers smartphone applications, online lecture notes, online exercises and projects, and electronic textbooks. It is appropriate for any STEM course with mathematical or computational topics. It improves STEM education by enhancing classroom teaching with ready-to-use demonstrations of mathematical concepts. Sage cells are easy to modify, allowing students to experiment and easily explore complicated concepts from any connected device, at any time.

Textbook authoring tools and the AIM Editorial Board contribute to society by making basic scientific knowledge more universally available and increasing participation by persons with disabilities. By creating, identifying and promoting high-quality open textbooks, we expand educational opportunities for all students, and in particular contribute to the creation of a diverse workforce by lowering the barrier for low-income students. It is possible to create output formats tailored for individuals with disabilities. For example, a first step is our webpage output with text, equations and graphics that all scale faithfully to large magnifications, making full content available to those with limited vision. Initial experiments by Alex Jordan suggest Nementh Braille is an easily achievable output format. We rely heavily on MathJax to render mathematics and that project is making real progress on math-to-speech.

The Sage Education Days workshops directly contribute to faculty development through training in the use of Sage cells, SageMathCloud and open textbooks in the classroom, and through the development of new course materials made freely available. Our workshops for authors and editorial work will increase the pool of faculty with experience using our tools. UTMOST will directly benefit the students at the test sites involved in this project.

Open textbooks and open software are publicly available and provide opportunities for motivated individuals to effectively study science and mathematics without formal enrollment, thus increasing public scientific literacy and public engagement with science and technology. The United States has the resources to be the leader in the distribution of educational material with open licenses across all academic disciplines. The ongoing work of UTMOST will continue to contribute to this leadership.

# 7 Results from Prior NSF Support

**Title, Award** Collaborative Research: UTMOST: Undergraduate Teaching of Mathematics with Open Software and Textbooks, CCLI Type II (Expansion), DUE-1022574, \$525,000, September 2010 to August 2013, with one-year no-cost extension to August 2014.

Results: Intellectual Merit Many new ideas resulted from our CCLI grant. The Sage Cell Server was developed by PI Grout and his students as an innovative way to provide Sage computations in a wide variety of situations. Towards the end of the grant, PI Stein began work on SageMathCloud, a huge undertaking motivated in part by our experiences with our test sites. PIs Judson and Beezer successfully created electronic versions of their textbooks, with interactive Sage components embedded. This work led to the design of MathBook XML for textbook authoring and exposed the desirability of the SL2X conversion tool.

The AIM Editorial Board has been very well received. Authors of open source textbooks now view a recommendation from the board as important for the success of their textbook projects. The board has also made more mathematics faculty aware of the existence of open textbooks and best practices when selecting an open textbook. Sage Educational Days workshops have proven to be a very popular format for exchanging pedagogical and technical information. Our test sites provided invaluable advice about the use of open software and open textbooks in their courses, and have provided quantifiable information via the Student Assessment of Learning Goals (SALG) surveys.

**Results: Broader Impacts** Approximately thirty faculty attended each of our Sage Educational Days workshops, and in turn they each teach many students and work with colleagues in their departments. We had formal relationships with eight mathematics departments as part of test sites program. Our test sites included participation from two Postsecondary Minority Institutions.

With open source software and textbooks readily available on the Internet, it is difficult to accurately estimate the magnitude of the impact. The SageMathCloud, despite being under heavy development, now hosts 140,000 accounts. The Sage Cell Server powers a wide variety of electronic resources and had 60,000 unique visits in the latest month. Beezer's online linear algebra textbook had 54,000 unique readers in October 2015, while Judson's online abstract algebra textbook had 10,500 readers for the same period.

**Selected Publications and Presentations** The first UTMOST grant has produced articles [5, 6, 11], talks [9, 10, 12, 67], and poster presentations [32, 33, 34, 35, 36].

Research Products Our research products include publicly accessible software: the Sage Cell Server [22, 23], SageMathCloud [64, 65], and MathBook XML [7,  $\langle\langle \text{Unresolved xref}, \text{ reference} \rangle$ " beezer-mathbook-source"; check spelling or use "provisional" attribute $\rangle\rangle$ ]. In addition we have produced Sage-enhanced textbooks [8, 31], and textbook evaluations [4].

## 8 Personnel

The UTMOST team includes mathematics faculty with decades of experience teaching undergraduates, open source textbook authors, software developers, mathematics education researchers and an experienced evaluator of STEM education initiatives.

- **Dr. Robert A. Beezer** Professor of Mathematics, University of Puget Sound. Beezer has 38 years teaching experience, is an open textbook author, and a Sage developer. Project Director.
- **Dr. David Farmer** Director of Programs, American Institute of Mathematics. At AIM he is responsible for the development of the AIM workshop style, and works with the team that runs 25 workshops and 40 SQuaREs per year. Principal Investigator.
- **Dr. Thomas W. Judson** Associate Professor of Mathematics, Stephen F. Austin State University. Judson has 38 years of teaching experience, is an open textbook author, and worked extensively with undergraduate mathematics teachers, and pre-service and inservice high school teachers. Principal Investigator.
- Ms. Susan Lynds Associate Scientist, University of Colorado. Lynds is a program evaluator at the Cooperative Institute for Research in Environmental Sciences where she has 10 years experience conducting internal and external evaluation studies for programs funded by the NSF, NOAA, NASA, the U.S. Departments of Education and Agriculture. External Evaluator.
- **Dr. Vilma Mesa** Associate Professor, School of Education and Department of Mathematics; Faculty Associate, Center for the Study of Higher and Postsecondary Education, University of Michigan. Mesa has degrees in mathematics and computer science, has worked for the Columbian Ministry of Economics managing the national tax network, and now teaches graduate courses in research methods and in mathematics education in the School of Education. An NSF CAREER award was used to explore mathematics teaching at community colleges. Principal Investigator.
- **Dr. Kent E. Morrison** Professor Emeritus, California Polytechnic State University, San Luis Obispo. Morrison is now affiliated with AIM, where he directs the AIM Editorial Board. He was department chair at Cal Poly, where he was involved in student retention, articulation with the community colleges, and career development for young faculty. Principal Investigator.
- **Dr. Theron J. Hitchman** Associate Professor, University of Northern Iowa. Hitchman has been active in the previous activities of the UTMOST Project, and is also very active in the Inquiry Based Learning (IBL) community. He will assist with the project's workshops. Senior Scientist.
- **Dr. Andrey Novoseltsev** Novoseltsov is a mathematician and software developer. He is the current volunteer maintainer of the Sage Cell Server. As a consultant to the project, he will maintain

and improve the Sage Cell Server, with guidance from the rest of the team.

- **Dr. Jason Grout** Scientific Software Developer, Bloomberg LP. Grout taught undergraduate mathematics at Drake University for several years, before joining the quantitative finance research division at Bloomberg to work on open source scientific software. He is on the Jupyter Steering Committee, and was a PI on the previous UTMOST grant. Advisory Board.
- **Dr. Kiran S. Kedlaya** Professor of Mathematics, University of California, San Diego. Kedlaya has research interests in number theory and algebraic geometry and has been very involved in the development and promotion of Sage for use in research and education. He was a PI on the previous UTMOST grant. Advisory Board.
- **Dr. Gavin LaRose** Lecturer IV; Instructional Technology Manager, University of Michigan. LaRose has 20 years of teaching experience and develops and manages the on-line homework and other instructional technology in the Department of Mathematics. Advisory Board.
- **Dr. William Stein** Professor of Mathematics, University of Washington; CEO and Founder, SageMath, Inc. Stein is also the founder and director of the Sage software project, in addition to designing and implementing SageMathCloud. He will closely advise consultants designing and implementing new features for SageMathCloud to support the project's activities. He was a PI on the previous UTMOST grant. Advisory Board.

School	Size	Type	State	Faculty	Spring 17	Fall 17	Spring 18	Fall 18
Cal State U, Long Beach George Washington U	36,000 10,000	Public Private	CA DC	Will Murray Dan Ullman		AA AA	AA AA	
George Washington C	10,000	Tilvate	DC	Murli Gupta		AA	AA	LA
Mississippi C	5,200	Religious	MS	John Travis			LA	
New York U	25,000	Private	NY	Drew Youngren	LA, P		LA	
				Trushant Majmudar	LA, CP			
Regis U	9,200	Religious	CO	Sally Duvall			LA, C	
				Megan Patnott				AA
Sam Houston State U	18,000	Public	TX	Brandy Doleshal		AA, C		
	<b>-</b> 000	D 111	3737	Malandro/Garcia	4.4. D			AA
State U of New York, Geneseo	5,300	Public	NY	Chris Leary	AA, P		AA	
Stephen F Austin State U	13,000	Public	TX	Matt Beauregard	LA, P			
Texas A&M U, Central Texas	2,300	Public	TX	Chris Thron			LA	
U of Michigan, Dearborn	7,300	Public	MI	Joan Remski		LA		LA
U of San Francisco	6,800	Religious	CA	Paul Zeitz				AA
U of Wisconsin, Oshkosh	12,600	Public	WI	Ken Price		LA	LA	LA

UTMOST Test-Site Schedule (LA: Linear Algebra, AA: Abstract Algebra, P: Pilot, C: Control)

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