All work on this assessment should be your own. The technology allowed on this test includes: Desmos (`desmos.com/calculator`) and a TI-84 calculator (or less). This exam has 9 questions, for a total of 100 points.

1. (50 points) Pick six (5) of the following ten (10) integrals and find the antiderivative. Specify the integration technique that you use. Please write as neatly as possible and clearly indicate your final answer.

   (a) $\displaystyle\int \frac{x}{1+x^2}\,dx$

   (b) $\displaystyle\int e^x \sin(x)\,dx$

(c) $\int (6x^2 - 8x)e^{x^3-2x^2+2}\,dx$

(d) $\int \sin^2(x)\cos^5(x)\,dx$

(e) $\int x \arctan(x)\, dx$

(f) $\int \dfrac{\sqrt{25 - x^2}}{x}\, dx$

(g) $\int x \ln(x)\, dx$

(h) $\int \dfrac{\sin(x)}{\cos^2(x) - \cos(x) - 2}\, dx$

(i) $\displaystyle\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

(j) $\displaystyle\int e^{-x}\cos(x)\,dx$

2. Determine if the following improper integrals converge or diverge. If it converges, state the value. If it diverges, show the limit that proves your claim. Do not use the comparison test for these problems.

(a) (5 points) $\displaystyle\int_1^\infty \frac{2x-3}{x^2-3x+4}\,dx$

(b) (5 points) $\displaystyle\int_{-\infty}^0 xe^x\,dx$

3. (10 points) Show that that $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \pi$.

4. (10 points) Let $I = \displaystyle\int_{1}^{\infty} \frac{1}{\sqrt{e^x - x}}\,dx$.

Circle the word that completes the following sentence: the integral in Question 4 _____.

CONVERGES / DIVERGES.

HINT: $\sqrt{e^x} \geq \sqrt[4]{e^x} \Rightarrow \dfrac{1}{\sqrt{e^x}} \leq \dfrac{1}{\sqrt[4]{e^x}}$

Proof:

5. Find the values of the following definite integrals. Leave your answer in exact form, i.e. no decimals answers.

(a) (10 points) $\displaystyle\int_0^1 \frac{x^3 - 2x^2 + 7x - 11}{x^2 + 1}\, dx$

(b) (10 points) $\displaystyle\int_2^3 \frac{1}{x^2\sqrt{9 - x^2}}\, dx$

Bonus Questions

You may attempt TWO of the following FOUR problems for a total of FOUR points extra credit. Only completely correct answers will be accepted, no partial credit will be awarded in this section.

1. $\int e^x \sqrt{\tan^2(e^x) + 1}\, dx$

2. Determine the values of a for which $\int_1^\infty \left(\frac{ax}{x^2+1} - \frac{1}{2x} \right) dx$ converges. Determine the value of the integrals for your specified a.

3. Consider the following function defined by an improper integral

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt.$$

Find the value of $\Gamma(3)$.

4. Find the value of $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{\sin^2(x) + 4\sin(x) + 4}\, dx$