

Your name:

Question 1 Find the limits of functions including the infinite limits, ∞ , $-\infty$. Otherwise write DNE. (Do not use L'hospital theorem.)

(1.6) a) $\lim_{x \rightarrow 5} (x^3 - 3x + 4)$

(2.4) n) $\lim_{\theta \rightarrow 0} \left[\frac{\cos \theta - 1}{2\theta^2} \right]$

(1.6) b) $\lim_{x \rightarrow -2} \left[\frac{x^3 + 2x^2 - 1}{5 - 3x} \right]$

(3.4) o) $\lim_{x \rightarrow \infty} \left[\frac{2x^2 - 7}{5x^2 + x - 3} \right]$

(1.6) c) $\lim_{x \rightarrow -1} \left[\frac{2x^2 + 3x + 1}{x^2 - 2x - 3} \right]$

(3.4) p) $\lim_{x \rightarrow -\infty} \left[\frac{x - 2}{x^2 + 1} \right]$

(1.6) d) $\lim_{h \rightarrow 0} \left[\frac{(2+h)^3 - 8}{h} \right]$

(3.4) q) $\lim_{x \rightarrow \infty} \left[\frac{x + 3x^2}{4x - 1} \right]$

(1.6) e) $\lim_{t \rightarrow 0} \left[\frac{\sqrt{t^2 + 4} - 2}{t^2} \right]$

(3.4) r) $\lim_{x \rightarrow \infty} \left[\frac{\sqrt{x + 3x^2}}{4x - 1} \right]$

(1.5) f) $\lim_{x \rightarrow 3^-} \frac{2x}{x - 3}$

(3.4) s) $\lim_{x \rightarrow -\infty} \left[\frac{\sqrt{x + 3x^2}}{4x - 1} \right]$

(1.5) g) $\lim_{x \rightarrow 3^+} \frac{2x}{x - 3}$

(3.4) t) $\lim_{x \rightarrow \infty} \left[\sqrt{9x^2 + x} - 3x \right]$

(1.5) h) $\lim_{x \rightarrow 0} \frac{1}{x^2}$

(3.4) u) $\lim_{x \rightarrow -\infty} \left[\sqrt{4x^2 + 3x} + 2x \right]$

(1.5) i) $\lim_{x \rightarrow 0} \frac{1}{x^3}$

(3.4) v) $\lim_{x \rightarrow -\infty} \left[\sqrt{4x^2 + 3x} - 2x \right]$

(1.6) j) $\lim_{x \rightarrow 1} \frac{|x - 1|}{x - 1}$

(3.4) w) $\lim_{x \rightarrow \infty} x \cdot \sin\left(\frac{2}{x}\right)$

(2.4) k) $\lim_{x \rightarrow 0} \left[\frac{\sin(5x)}{3x} \right]$

(3.4) x) $\lim_{x \rightarrow \infty} \sqrt{x} \cdot \sin\left(\frac{2}{x}\right)$

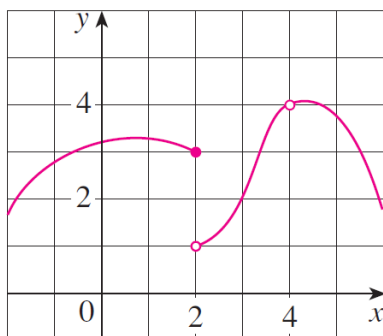
(2.4) l) $\lim_{\theta \rightarrow 0} \left[\frac{\cos \theta - 1}{\sin \theta} \right]$

(2.2, honor) y) $\lim_{x \rightarrow 1} \left[\frac{x^{2015} - 1}{x - 1} \right]$

(2.4) m) $\lim_{x \rightarrow 0} \left[\frac{\sin(3x) \sin(5x)}{x^2} \right]$

(4.3, honor) z) $\lim_{h \rightarrow 0} \frac{1}{h} \int_2^{2+h} \sqrt{1 + t^3} dt$

Question 2 (1.5) Use the given graph of f to state the value of each quantity, if exists. If it does not exist, explain why.



a) $\lim_{x \rightarrow 2^-} f(x)$

b) $\lim_{x \rightarrow 2^+} f(x)$

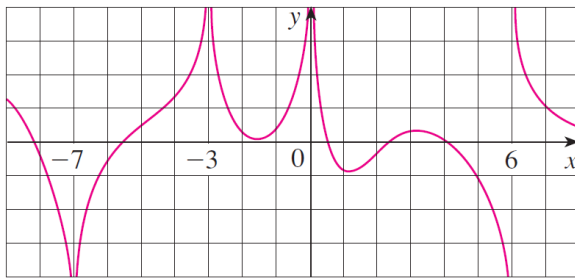
c) $\lim_{x \rightarrow 2} f(x)$

d) $f(2)$

e) $\lim_{x \rightarrow 4} f(x)$

f) $f(4)$

Question 3 (1.5) For the function f whose graph is shown, state the following.

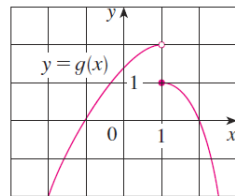
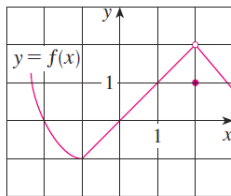


- $\lim_{x \rightarrow -7} f(x)$
- $\lim_{x \rightarrow -3} f(x)$
- $\lim_{x \rightarrow 0} f(x)$
- $\lim_{x \rightarrow 6^-} f(x)$
- $\lim_{x \rightarrow 6^+} f(x)$
- The equations of the vertical asymptote(s).

Question 4 (1.6) Using squeeze theorem, show

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

Question 5 (1.6) The graphs of f and g are given. State the following. Use $\infty, -\infty$ for infinite limits. If the limit does not exist or is not infinite limits, explain why.



- $\lim_{x \rightarrow 1^-} (f(x) + g(x))$.
- $\lim_{x \rightarrow -1} \frac{f(x)}{g(x)}$.
- $\lim_{x \rightarrow 1} \sqrt{3 + f(x)}$.

Question 6 (1.6) a) What is wrong with the following equation?

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

b) Explain why the following equation is correct.

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} (x + 3)$$

Question 7 (1.6,1.8) Let

$$f(x) = \begin{cases} x^2 + 1 & , x < 1 \\ (x - 2)^2 & , x \geq 1 \end{cases}$$

- Find $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$
- Does $\lim_{x \rightarrow 1} f(x)$ exist?
- Is f continuous at $x = 1$? Explain.
- Sketch the graph f .

Question 8 (1.6,1.8) Let

$$f(x) = \frac{x^2 + x - 6}{|x - 2|}$$

- Find $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$
- Does $\lim_{x \rightarrow 2} f(x)$ exist?
- Is f continuous at $x = 2$? Explain.

d) Sketch the graph f .

Question 9 (1.6) If $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1} = 10$, find $\lim_{x \rightarrow 1} f(x)$.

Question 10 (1.8) Explain why the function f is discontinuous at the given number a .

a) $a = 0$

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$$

b) $a = 1$

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

Question 11 (1.8) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

Question 12 (1.8) Use IVT to show there is a root of the given equation in the specified interval.

a)

$$x^4 + x - 3 = 0, \quad (1, 2)$$

b) Use the calculator.

$$\sin x = x^2 - x, \quad (1, 2)$$

(honor) c) Use the calculator.

$$\sqrt{x - 5} = \frac{1}{x + 3}, \quad (-\infty, \infty)$$

Question 13 (1.6, 1.8, honor) For what values of x is f continuous?

a)

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

b)

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$$

Question 14 Find derivatives.

(2.3) a) $f(x) = 2^{12}$

(2.3) b) $f(x) = 3x^2 - 4x + 1$

(2.3) c) $f(x) = \frac{1}{2}x - \frac{1}{3}$

(2.3) d) $f(x) = (x - 2)(2x + 3)$

(2.3) e) $f(x) = (1 + x + x^2)(2 - x^4)$

(2.3) f) $f(x) = \frac{2x + 1}{x + 4}$

(2.3) g) $f(x) = \frac{x^2 - 1}{2x - 3}$

(2.3) h) $f(x) = \frac{x}{(x - 1)^2}$

(2.3) i) $f(x) = \frac{x}{x^2 - 1}$

(2.3) j) $f(x) = x^{-2}$

(2.3) k) $f(x) = \sqrt{x} - x$

(2.3) l) $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$

(2.3) m) $f(x) = \frac{\sqrt{x} + x}{x^2}$

(2.3) n) $f(x) = \frac{4}{1 - x}$

(2.4) o) $f(x) = 3x^2 - 2 \cos x$

(2.4) p) $f(x) = \sec x \cdot \tan x$

(2.4) q) $f(x) = \frac{\tan x - 1}{\sec x}$

(2.4) r) $f(x) = \sqrt{x} \sin x$

(2.5) s) $f(x) = \sin(\sqrt{x})$

(2.5) t) $f(x) = \sqrt{\sin x}$

(2.5) u) $f(x) = (2x^3 + 5)^4$

(2.5) v) $f(x) = \sqrt{1 - 2x}$

(2.5) w) $f(x) = \sqrt{x^2 + 4}$

(2.5) x) $f(x) = \sqrt{x + \sqrt{x}}$

(2.5) y) $f(x) = \sec^2 x + \tan^2 x$

(2.5) z) $f(x) = \sin(x^2) + \cos x$

(2.5) aa) $f(x) = \sin(\tan(2x))$

Question 15 Find $f'(x)$ using the **definition of the derivative**.

(2.1) a) $f(x) = 3x^2 - 4x + 1$

(2.1) b) $f(x) = \frac{1}{x}$

(2.1) c) $f(x) = 2\sqrt{x}$

(2.4) d) $f(x) = \sin x$

Question 16 (2.3) Suppose that $f(4) = 2$, $g(4) = 5$, $f'(4) = 6$, $g'(4) = -3$. Find $h'(4)$.

a) $h(x) = 3f(x) + 8g(x)$

c) $h(x) = \frac{f(x)}{g(x)}$

b) $h(x) = f(x)g(x)$

d) $h(x) = \frac{g(x)}{f(x)+g(x)}$

Question 17 (2.3) If $h(2) = 4$ and $h'(2) = -3$, find

$$\left. \frac{d}{dx} \left(\frac{h(x)}{x} \right) \right|_{x=2}$$

Question 18 (2.5) If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.

Question 19 Find an equation of the tangent line to the curve at the given point.

(2.3) a)

$$y = x^4 + 2x^2 - x, (1, 2)$$

(2.3) c)

$$y = \frac{\sqrt{x}}{1 + x^2}, \left(1, \frac{1}{2}\right)$$

(2.3) b)

$$y = \frac{2x}{x + 1}, (1, 1)$$

(2.4) d)

$$y = \sec x, \left(\frac{\pi}{3}, 2\right)$$

(2.5) e)

$$y = \sqrt{1+x^3}, (2,3)$$

(2.6) f)

$$x^2 + xy + y^2 = 3, (1,1)$$

Question 20 (2.6) Find $\frac{dy}{dx}$.

a)

$$x^2 + xy - y^2 = 4$$

c)

$$xy = \sqrt{x^2 + y^2}$$

b)

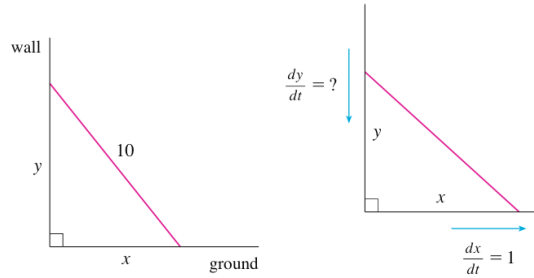
$$x^4 + x^2y^2 + y^3 = 5$$

(honor) d)

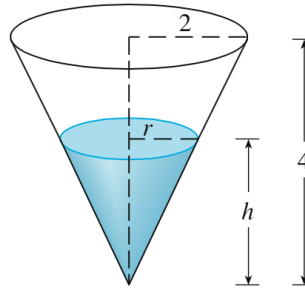
$$\tan\left(\frac{x}{y}\right) = x + y$$

Question 21 (2.8) Air is being pumped into a spherical balloon so that its volume increase at a rate of $100\text{cm}^3/\text{s}$. How fast is the radius of the balloon increasing when the diameter is 50cm . Hint: $V = \frac{4\pi r^3}{3}$.

Question 22 (2.8) A ladder 10ft long rest against a vertical wall. If the bottom of the ladder slide away from the wall at a rate 1ft/s , how fast is the top of the latter sliding down the wall when the botton of the ladder is 6ft from the wall?



Question 23 (2.8) A water tank has the shape of an inverted circular cone with base radius 2m and height 4m . If water is being pumped into the tank at a rate of $2\text{m}^3/\text{min}$, find **the rate at which the water level is rising** when the water is 3m deep. Hint: $V = \frac{1}{3}\pi r^2h$.



Question 24 (2.1, honor) Determine whether $f'(0)$ exists.

a)

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

b)

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Question 25 (2.2, honor) Let

$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$$

Is f differentiable at 1 ?

Hint: Use

$$f'_-(a) := \lim_{h \rightarrow a^-} \frac{f(h+a) - f(a)}{h}, \text{ Left-hand derivative of } f$$

$$f'_+(a) := \lim_{h \rightarrow a^+} \frac{f(h+a) - f(a)}{h}, \text{ Right-hand derivative of } f$$

and

$$f'(a) \text{ exists} \Leftrightarrow f'_+(a) = f'_-(a)$$

Question 26 (2.3) Let

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ mx + b, & x > 2 \end{cases}$$

Find the value of m and b that make f differentiable everywhere.

Question 27 (2.4) Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

a) $\frac{d^{99}}{dx^{99}}(\sin x)$

(honor) b) $\frac{d^{30}}{dx^{30}}(x \cdot \sin x)$

Question 28 (2.8) Suppose $y = \sqrt{2x+1}$ where x and y is functions of t .

- If $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = 4$.
- If $\frac{dy}{dt} = 5$, find $\frac{dx}{dt}$ when $x = 12$.

Question 29 (2.8) Suppose $4x^2 + 9y^2 = 36$ where x and y is functions of t .

- If $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = -2$ and $y = \frac{2\sqrt{5}}{3}$.
- If $\frac{dy}{dt} = \frac{1}{3}$, find $\frac{dx}{dt}$ when $x = 2$ and $y = \frac{2\sqrt{5}}{3}$.

Question 30 (2.9) Find the linearization $L(x)$ of the function f at a where

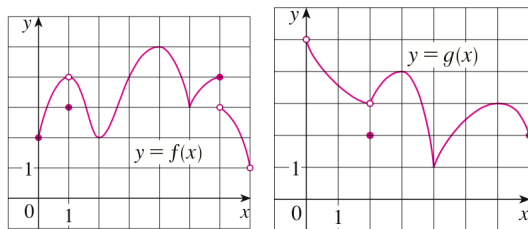
$$L(x) = f(a) + f'(a)(x - a)$$

- $f(x) = x^4 + 3x^2$, $a = -1$
- $f(x) = \sqrt{x}$, $a = 4$

Question 31 (2.9) Find the differential dy of each function.

- $y = x^2 \cdot \sin 2x$
- $y = \frac{s}{1+2s}$

Question 32 (3.1) Use the graph to state the absolute and local maximum and minimum values and corresponding points of the function f and g . **Note:** By Stewart's definition, endpoints cannot be a local maximum or minimum.



Question 33 (3.1) Find the the critical number(s) of the function f .

- a) $f(x) = 2x^3 - 3x^2 - 36x$.
- b) $f(x) = |3x - 4|$.
- c) $f(x) = \frac{x-1}{x^2+4}$.
- d) $f(x) = 2 \cos x + \sin^2 x$.

Question 34 (3.1) Find the the absolute maximum and minimum values of the function f .

- a) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$.

b) $f(x) = 3x^4 - 16x^3 + 18x^2, [-1, 4]$.

c) $f(x) = x\sqrt{4-x^2}, [-1, 2]$.

Question 35 (3.2) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval, Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 - 3x + 2, [-2, 2]$$

Question 36 (3.2) Show that the equation $x^3 - 12x + 1 = 0$ has exact one root in the interval $[-1, 1]$.

HINT: For the existence, use IVT. For the uniqueness, use Rolle's Theorem.

Question 37 (3.3,3.4) Given f ,

a) $f(x) = 2x^3 + 3x^2 - 36x$

b) $f(x) = 3x^4 - 8x^3 - 24x^2 + 96x$

c) $f(x) = \frac{x}{x^2+1}$

- Find the intervals on which f is increasing. (Exclude endpoint(s) if exist(s).)
- Find the intervals on which f is decreasing. (Exclude endpoint(s) if exist(s).)
- Find the local minimum value(s).
- Find the local maximum value(s).
- Find the intervals on which f is concave upward.
- Find the intervals on which f is concave downward.
- Find (an) inflection point(s) if exists.

Question 38 (3.4) Sketch the curve.

a) $y = \frac{2x^2}{x^2-1}$

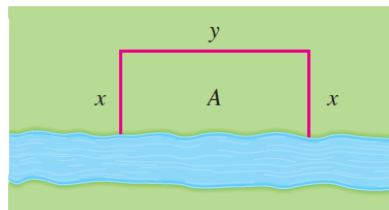
c) $y = \frac{x^2}{\sqrt{x+1}}$

b) $y = \frac{x}{x^2-9}$

d) $y = \frac{x}{\sqrt{x^2+1}}$

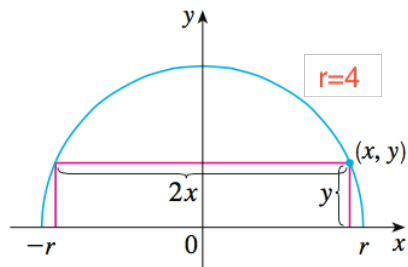
- Find the domain.
- Find x -and y -intercepts.
- Find the symmetry if exists.(even, odd, periodic).
- Find the asymptote(s).
- Find the interval(s) of Increase or Decrease.
- Find the local maximum and minimum value(s).
- Find concavity and points of inflection.
- Sketch the Curve.

Question 39 (3.7) A farmer has 2400ft of fencing and wants to fence off a rectangular file that border a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?



Question 40 (3.8) Find the area of the largest rectangle that can be inscribed in a semicircle of radius $r = 4$.

Hint: Describe y in terms of x . $0 \leq x \leq 4$, $0 \leq y \leq 4$.



Question 41 (3.9) Find the most general anti-derivative of each of the following functions.

(3.9) a) $f(x) = \frac{1}{2}x^2 - 2x + 6$

(4.4) i) $f(x) = 2\sqrt{x} + 2\sec(x)\tan(x)$

(3.9) b) $f(x) = (x - 1)^2$

(4.4) j) $f(x) = \sqrt{x} + 2\sin(x) + x$

(3.9) c) $f(x) = \frac{10}{x^9}$

(4.5) k) $f(x) = x^2\sqrt{x^3 + 1}$

(3.9) d) $f(x) = \sqrt{2}$

(4.5) l) $f(x) = (1 - 2x)^9$

(3.9) e) $f(x) = \frac{3}{\sqrt{x}}$

(4.5) m) $f(x) = (x^2 + 1)(x^3 + 3x)^4$

(3.9) f) $f(x) = (x + 1)(2x - 1)$

(4.5) n) $f(x) = \sec^3(x)\tan(x)$

(3.9) g) $f(t) = \frac{1+t+t^2}{\sqrt{t}}$

(4.5) o) $f(x) = x\sqrt{1+x^2}$

(3.9) h) $f(x) = \sqrt{\frac{3}{x}}$

(4.5) p) $f(x) = x^3\sqrt{1+x^2}$

Question 42 (3.9) Find the function f if

a) $f'(x) = 5x^4 - 2x^5$ and $f(0) = 4$.

b) $f''(x) = \sin(x) + \cos(x)$, $f(0) = 3$ and $f'(0) = 4$.

Question 43 (4.2) Express the limit as a definite integral on the given interval.

a)

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1 - x_i^2}{4 + x_i^2} \cdot \Delta x, [2, 6]$$

b)

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}, [0, 1]$$

Question 44 (4.2) Using the **the definition of the (definite) integral** to evaluate the integral.

a) $\int_2^5 (4 - 2x)dx$

b) $\int_0^1 x^3 dx$

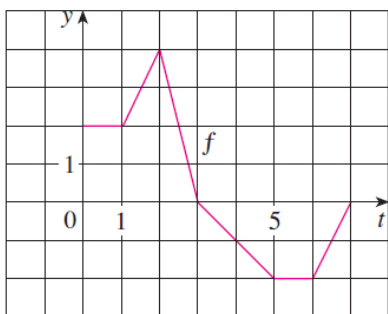
Question 45 (4.2) If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]dx$.

Question 46 (4.2) If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.5$, find $\int_1^4 2f(x)dx$.

Question 47 (4.2) Use the properties of integrals to verify the inequality.

$$2 \leq \int_{-1}^1 \sqrt{1+x^2} dx \leq 2\sqrt{2}$$

Question 48 (4.3) Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.



- Evaluate $g(0), g(1), g(2), g(3), g(6)$.
- On what interval is g increasing?
- Where g have a maximum value?
- Sketch a rough graph of g .

Question 49 (4.3) Use the fundamental theorem of Calculus to find the **derivative** of the function.

- $f(x) = \int_1^x \frac{1}{t^3+1} dt$
- $f(x) = \int_x^\pi \sqrt{1 + \sec(t)} dt$
- $f(x) = \int_0^{x^4} \cos^2(\theta) d\theta$
- $f(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^4+1} dt$

Question 50 Evaluate.

(4.3) a) $\int_{-1}^2 (x^3 - 2x) dx$

(4.5) g) $\int_1^4 \sqrt{\frac{5}{x}} dx$

(4.3) b) $\int_{\frac{\pi}{6}}^\pi \sin \theta d\theta$

(4.5) h) $\int_0^{\frac{\pi}{2}} \cos(x) \sin(\sin(x)) dx$

(4.3) c) $\int_1^2 (1 + 2y)^2 dy$

(4.5) i) $\int_1^2 x\sqrt{x-1} dx$

(4.4) d) $\int_0^2 |2x - 1| dx$

(4.5) j) $\int_0^1 x^3 \sqrt{1+x^2} dx$

(4.4) e) $\int_{-1}^2 (x - 2|x|) dx$

(4.4) k) $\int_1^9 \frac{x-1}{\sqrt{x}} dx$

(4.5) f) $\int_{-1}^1 x(1-x)^2 dx$

(4.4) l) $\int_{-2}^2 f(x) dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \leq x \leq 0 \\ 4 - x^2 & \text{if } 0 < x \leq 2 \end{cases}$

Question 51 (4.4) What is wrong with the equation? Explain.

$$\int_{-2}^1 x^{-4} dx = \left. \frac{x^{-3}}{-3} \right|_{-2}^1 = -\frac{3}{8}$$

Question 52 (4.3) Evaluate the limit by recognizing the sum as a Riemman sum.

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^3 + \left(\frac{2}{n}\right)^3 + \dots + \left(\frac{n}{n}\right)^3 \right]$$

Question 53 (4.3) Evaluate the limit by recognizing the sum as a Riemman sum.

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \dots + \sqrt{\frac{n}{n}} \right)$$

Question 54 (4.4) The velocity function (in meters per second) is given for a particle moving along a line.

$$v(t) = 3t - 5, 0 \leq t \leq 3$$

- Find the displacement

- Find the distance traveled.

Question 55 (4.5) If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.

Question 56 (4.5) If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx$.

Question 57 (5.1) Find the area of the region enclosed by the parabola $y = 5x - x^2$ and the line $y = x$.

- Find the points of intersection.
- Find the area.

Question 58 (5.1) Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

- Find the points of intersection.
- Find the area.