Does an odd perfect number exist ?

Hafsa ELIBRAHIMI

Claim that M is an odd perfect number

The prime factor decomposition of M is

$$M = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots.\ p_r^{\alpha_r}$$

where $p_1, p_2, p_3, \ldots., p_r$ are prime numbers

and $\alpha_1, \alpha_2, \alpha_3, \ldots., \alpha_r$ are natural numbers

Since M is an odd perfect number

Then $M = 1 + K$

where K is the sum of all the divisors of M except 1 and M itself

We know that M is odd and 1 is odd

Thus K must be even (because odd + even = odd)

On the other hand, $p_1, p_2, p_3, \ldots., p_r \neq 2$

(because M is odd)

Therefore, all the terms that constitute K are odd

Let H be the number of terms that constitute K must be even

Thus H is even (in order to have K even for example 5+3+1+7 = 16 even)

We know that the number of divisors of M is

$(\alpha_1+1) \times (\alpha_2+1) \times (\alpha_3+1) \; .... \times (\alpha_r+1)$

Then H = $(\alpha_1+1) \times (\alpha_2+1) \times (\alpha_3+1) \; .... \times (\alpha_r+1) - 2$ (we remove 1 and M itself)

Since H is even and 2 is even

$(\alpha_1+1) \times (\alpha_2+1) \times (\alpha_3+1) \; .... \times (\alpha_r+1)$ must be even

(because odd - odd = even)

- We conclude that :

At least one of (α_i+1)must be even

Since 1 is odd then α_i must be odd

- If an odd perfect number exist than he must have at least one odd exponent in his prime factor decomposition

- In contrast,

if $(\alpha_1+1) \times (\alpha_2+1) \times (\alpha_3+1) \; .... \times (\alpha_r+1)$ is odd

then all α_i are even (because 1 is odd)

Thus, the number can’t be a perfect number

For example :

$9 = 3^2$ can’t be a perfect number

$9\ 765\ 625 = 5^{10}$ can’t be a perfect number