

Proyecto Mate 4 P1 (Saraí Perez)

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```
# Let us first find the characteristic equation and solve it:
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```
var('r')  
solve([(r-2)^3*(r^2 + 9)== 0], r)
```

```
r
```

```
[r == (-3*I), r == (3*I), r == 2]
```

```
# Among the roots from the characteristic equations that were found,\  
there is one with multiplicity 3 and this one will be used to \  
find the complementary solution. Let's start with the particular \  
one:
```

```
#We have multiplied everything times x to take out the duplicity.
```

```
var('x,A,B,C,D,E,F,G')  
yp=(A+B*x+C*x^2)*exp(x)+(D*x+E*x^2)*cos(3*x)+(F*x+G*x^2)*sin(3*x)  
show(yp)
```

```
(x, A, B, C, D, E, F, G)
```

```
( $E x^2 + D x$ )  $\cos(3 x)$  + ( $C x^2 + B x + A$ )  $e^x$  + ( $G x^2 + F x$ )  $\sin(3 x)$ 
```

```
#Now, we will substitute the previous equation into the original \  
differential equation:
```

```
rem0= diff(yp,x,2)+9*yp  
rem1= diff(rem0,x)-2*rem0  
rem2= diff(rem1,x)-2*rem1  
rem3= diff(rem2,x)-2*rem2  
remfinal= rem3-x^2*exp(x)-x*sin(3*x)  
show(remfinal)
```

```
-x^2 e^x - 54 (2 E x + D)  $\cos(3 x)$  + 276 (2 G x + F)  $\cos(3 x)$  + 524 E  $\cos(3 x)$  - 162 G  $\cos(3 x)$  -  
10 (C x^2 + B x + A) e^x + 28 (2 C x + B) e^x - 50 C e^x - 276 (2 E x + D)  $\sin(3 x)$  - 54 (2 G x + F)  $\sin(3 x)$  +  
162 E  $\sin(3 x)$  + 524 G  $\sin(3 x)$  - x  $\sin(3 x)$ 
```

```
#Now, we will find the coefficients of the previous equation
```

```
coeffEx=remfinal.coefficient(exp(x))  
coeffCos=remfinal.coefficient(cos(3*x))
```

```
coeffSin=remfinal.coefficient(sin(3*x))
```

```
#We will use the command Solve to find the coefficients A, B, C.
solve([coeffEx.coefficient(x,0)==0,coeffEx.coefficient(x,1)==0,
coeffEx.coefficient(x,2)==0],A,B,C)
[[A == (-267/250), B == (-14/25), C == (-1/10)]]
```

```
#And now, we will find the remaining coefficients of the problem, \
which are the coefficients D, E, F, G.
solve([coeffCos.coefficient(x,0)==0,coeffCos.coefficient(x,1)==0,
coeffSin.coefficient(x,0)==0,coeffSin.coefficient(x,1)==0],D,E,F,G)
[[D == (-251/114244), E == (-23/13182), F == (1379/514098), G == (-3/8788)]]
```

```
#In short, the values of the coefficients found are the following \
ones:
```

```
A=-267/250; B=-14/25; C=-1/10; D=-251/114244; E=-23/13182; F\
=1379/514098; G=-3/8788;
```

```
#We are now finally able to find the complementary solution as \
follows:
```

```
var('c1, c2, c3, c4, c5')
yc=c1*exp(2*x)+c2*x*exp(2*x)+c3*x^2*exp(2*x)+c4*cos(3*x)+c5*sin(3*x)\
+(A+B*x+C*x^2)*exp(x)+(D*x+E*x^2)*cos(3*x)+(F*x+G*x^2)*sin(3*x)
show(yc)
```

```
(c1, c2, c3, c4, c5)
```

$$c_3 x^2 e^{(2x)} + c_2 x e^{(2x)} - \frac{1}{342732} (598 x^2 + 753 x) \cos(3x) + c_4 \cos(3x) + c_1 e^{(2x)} - \frac{1}{250} (25 x^2 + 140 x + 267) e^x - \frac{1}{1028196} (351 x^2 - 2758 x) \sin(3x) + c_5 \sin(3x)$$

```
#Solving for the coefficients c1, c2, c3, c4, c5 we find:
```

```
solve([yc.substitute(x=0)==1,diff(yc,x).substitute(x=0)==1,diff(yc,x)\
,2).substitute(x=0)==1,diff(yc,x,3).substitute(x=0)==1,diff(yc,x)\
,4).substitute(x=0)==1],c1,c2,c3,c4,c5)
```

```
[[c1 == (774782/371293), c2 == (-43330/28561), c3 == (1020/2197), c4 ==
(-1737019/92823250), c5 == (-4850123/556939500)]]
```

```
#In short, the values of the coefficients c1, c2, c3, c4, c5 are:
c1 =774782/371293; c2 = -43330/28561; c3 =1020/2197; c4= \
-1737019/92823250; c5 = -4850123/556939500
```

```
#And my complete solution (complementary and particular)for the \
differential equation with the initial conditions given, is:
```

```
yc=c1*exp(2*x)+c2*x*exp(2*x)+c3*x^2*exp(2*x)+c4*cos(3*x)+c5*sin(3*x)\
+(A+B*x+C*x^2)*exp(x)+(D*x+E*x^2)*cos(3*x)+(F*x+G*x^2)*sin(3*x)
show(yc)
```

$$\frac{1020}{2197} x^2 e^{(2x)} - \frac{1}{342732} (598 x^2 + 753 x) \cos(3x) - \frac{43330}{28561} x e^{(2x)} - \frac{1}{250} (25 x^2 + 140 x + 267) e^x - \frac{1}{1028196} (351 x^2 - 2758 x) \sin(3x) - \frac{1737019}{92823250} \cos(3x) + \frac{774782}{371293} e^{(2x)} - \frac{4850123}{556939500} \sin(3x)$$