3. Suppose $p_1 \cdots p_n$ are all the primes of the form $6x-1$.
let $N = 6p_1 \cdots p_n - 1$. Note that $p_i \nmid N$ for all i.
Also note that 2 and 3 do not divide N.
The primes ≥ 5 are of the form $6x-1$ or $6x+1$ since none of $6x+2$, $6x+3$, and $6x+4$ can be prime ≥ 5.
If every divisor of N is of the form $6x+1$, then N is also of the form $6x+1$.
This is a contradiction since N is of the form $6x-1$.

4(a) By the Division Algorithm, any integer n can be writen n in the form $4q+r$ for some integers q and r, where $r=0,1,2,$ or 3.

1. $r=0$

$$n^2 = (4q+0)^2 = 16q^2 = 4(4q^2)$$

2. $r=1$

$$n^2 = (4q+1)^2 = 16q^2+8q+1 = 4(4q^2+2q)+1$$

3. $r=2$

$$n^2 = (4q+2)^2 = 16q^2+16q+4 = 4(4q^2+4q+1)$$

We see that n^2 is of the form $4k$ or $4k+1$ for some integer k.
Hence there is no perfect square of the form $4k+3$.

(b) Suppose that one of the elements in the sequence is a perfect square.
let n denote the root of that element.
The unit digit of n must be either 1 or 9.

- $\nexists n \in \{01, 11, 21, 31, 41, 51, 61, 71, 81, 91\}$ s.t that the last 2 digits of n^2 are 11.
- $\nexists n \in \{09, 19, 29, 39, 49, 59, 69, 79, 89, 99\}$ s.t. that the last 2 digits of n^2 are 11

Any other digits of n surely have no impact on these last 2 digits.
Therefore no element in the squence is a perfect square.

gcd(455, 1235) = 65

$$\begin{array}{r} 2 \\ 455\overline{)1235} \\ \underline{910} \\ 325 \end{array} \quad \begin{array}{r} 1 \\ 325\overline{)455} \\ \underline{325} \\ 130 \end{array} \quad \begin{array}{r} 3 \\ 130\overline{)455} \\ \underline{390} \\ \boxed{65} \end{array} \quad \begin{array}{r} 7 \\ 65\overline{)455} \\ \underline{455} \\ 0 \end{array}$$

Find primes up to 150.

For all number a : from 2 to sqrt(n)

If a is unmarked then a is prime.

for all multiples of a(a<n)

mark multiples as composite

All unmaked numbers are prime.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150