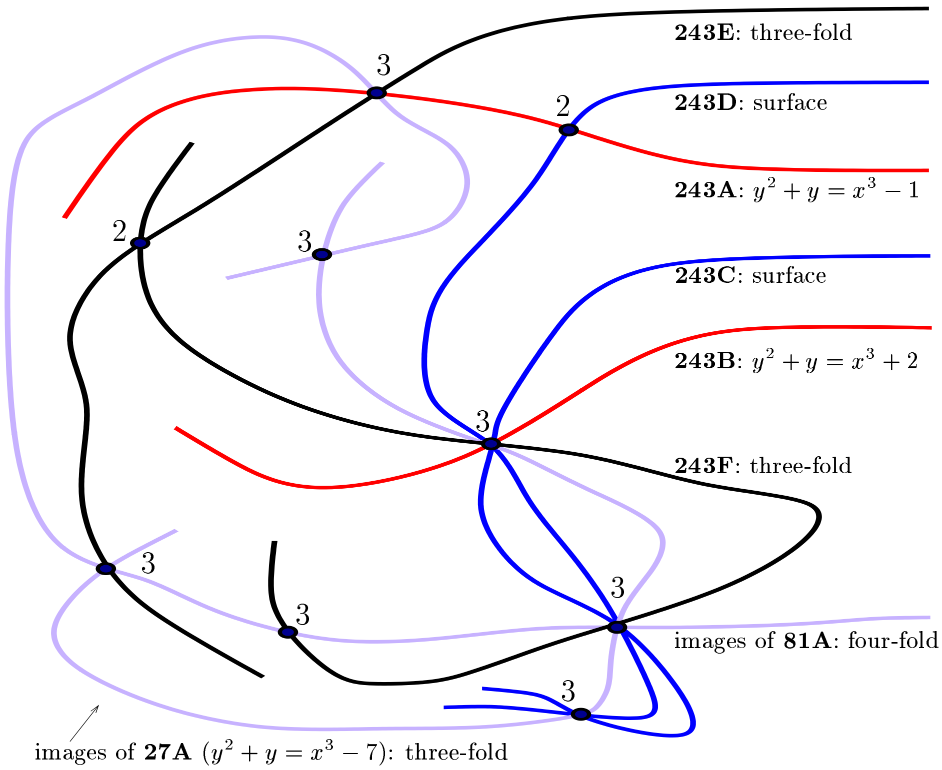
Modularity of Elliptic Curves and Beyond
MSRI 1999

Modularity theorem: *Every elliptic curve over* $\mathbf{Q}$ *is modular.*

The proof follows a program initiated by R. Taylor and A. Wiles. For further details see C. Breuil, B. Conrad, F. Diamond, and R. Taylor, *On the modularity of elliptic curves over* $\mathbf{Q}$.



The Jacobian of $X_0(243)$

Artin's conjecture: The L-series of any continuous representation $\mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{GL}_n(\mathbf{C})$ is entire, except possibly at 1.

Results:

— E. Artin, *Über eine neue Art von L-Reihen*, Abh. Math. Sem. Univ. Hamburg **3** (1924), 89–108.

— J. Buhler, *Icosahedral Galois representations*, LNM 654, 1978.

— K. Buzzard, M. Dickinson, N. Shepherd-Barron, and R. Taylor, *On icosahedral Artin representations*.

— G. Frey, *On Artin's conjecture for odd 2-dimensional representations*, LNM 1585, 1994.

— E. Hecke, *Eine neue Art von Zetafunktionen und ihre Beziehungen zur Verteilung der Primzahlen*, Math. Z. **6** (1920), 11–51.

— R. P. Langlands, *Base change for* GL(2), Princeton University Press, Princeton, 1980.

— J. Tunnell, *Artin's conjecture for representations of octahedral type*, Bull. AMS **5** (1981), 173–175.