Worksheet 1/29/2020

1. $\int (x-2)\sin(x)\ dx = (2-x)\cos(x) + \int \cos(x)\ dx = \boxed{(2-x)\cos(x) + \sin(x) + C}$.

$u = x-2$	$du = dx$
$v = -\cos(x)$	$dv = \sin(x)\ dx$

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2. $\int x^2 \ln(x)\ dx = \frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^2\ dx = \boxed{\frac{x^3}{3}\ln(x) - \frac{x^3}{9} + C}$.

$u = \ln(x)$	$du = \frac{1}{x}\ dx$
$v = \frac{1}{3}x^3$	$dv = x^2 dx$

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3. $\int x\sqrt{x+1}\ dx = \int (u-1)\sqrt{u}\ du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\ du = \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} = \boxed{\frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} + C}$.

$u = x+1$

$x = u+1$

$dx = du$

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4. $\int x^5\sqrt{x^3+1}\ dx = \frac{1}{3}\int x^3\sqrt{u}\ du = \frac{1}{3}\int u\sqrt{u+1}\ du = \frac{1}{3}\left(\frac{2(u+1)^{\frac{5}{2}}}{5} - \frac{2(u+1)^{\frac{3}{2}}}{3}\right) = \boxed{\frac{2(x^3+1)^{\frac{5}{2}}}{15} - \frac{2(x^3+1)^{\frac{3}{2}}}{9} + C}$

$u = x^3$

$du = 3x^2 dx$

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5. $\int x^2\sin(x)dx = -x^2\cos(x) + \int \cos(x)2x\ dx = -x^2\cos(x) + \left(2x\sin(x) - \int 2\sin(x)\ dx\right) = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) = \boxed{\cos(x)(2-x^2) + \sin(x) + C}$.

$u = x^2$	$du = 2x\ dx$	$w = 2x$	$dw = -sin(x)\ dx$
$v = -\cos(x)$	$dv = \sin(x)\ dx$	$z = e^x$	$dz = e^x\ dx$

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6. $I = \int \sin(x)e^x dx = \sin(x)e^x - \int \cos(x)e^x\ dx = \sin(x)e^x - (\cos(x)e^x + \int \sin(x)e^x\ dx) = e^x(\sin(x) - \cos(x)) - I$.

$u = \sin(x)$	$du = \cos(x)\ dx$	$w = \cos(x)$	$dw = -\sin(x)\ dx$
$v = e^x$	$dv = e^x\ dx$	$z = e^x$	$dz = e^x\ dx$

$\implies 2I = e^x(\sin(x) - \cos(x)) \implies I = \boxed{\frac{e^x(\sin(x) - \cos(x))}{2} + C}$.